STADIRA
TRAINING INSTITUTE

Mathematics – Analysis and Approaches
Higher Level
Paper 1 – Marking Key

Marks: 110
2 hours

DP MOCK EXAMINATION – SET 1

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Instructions to candidates

- This marking key uses IB-style notation: M = method mark, A = accuracy mark, R = reasoning mark, AG = answer given, FT = follow through from previous part.
- Award FT marks where a candidate uses an incorrect earlier answer correctly in later working.
- This paper is NON-CALCULATOR: answers must be exact (fractions, surds, multiples of π). A correct decimal approximation without the exact value does not earn the final accuracy mark.
- No GDC methods appear in this key, and no credit is available for calculator notation.

Information

- This marking key has 8 pages.
- The number of marks for each question or part question is shown in brackets [ ].

Question 1

Question	Solution / Working	Marks	Notes
1	$\dfrac{7x+1}{(x-1)(x+3)} = \dfrac{A}{x-1} + \dfrac{B}{x+3}$	M1	Equating coefficients is equally valid: A + B = 7, 3A − B = 1
	$7x+1 = A(x+3) + B(x-1)$	A1	
	$x = 1:\ 8 = 4A \implies A = 2$	A1	
	$x = -3:\ -20 = -4B \implies B = 5$	A1	
	$\dfrac{7x+1}{(x-1)(x+3)} = \dfrac{2}{x-1} + \dfrac{5}{x+3}$	A1	

Total: [5 marks]

Question 2

Question	Solution / Working	Marks	Notes
2(a)	$\sin 75° = \sin(45° + 30°)$	M1	Must use 45° + 30° (or equivalent standard angles); exact surd values required
	$= \sin 45° \cos 30° + \cos 45° \sin 30°$		
	$= \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2}$	A1	
	$= \dfrac{\sqrt{6}+\sqrt{2}}{4}$ AG	AG1	
2(b)	$\tan 105° = \tan(60° + 45°) = \dfrac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$	M1	Or tan 105° = −cot 15° routes; rationalising step must be seen
	$= \dfrac{\sqrt{3}+1}{1-\sqrt{3}} = \dfrac{(\sqrt{3}+1)(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = \dfrac{4+2\sqrt{3}}{-2}$	A1	
	$= -(2+\sqrt{3})$ AG	AG1	

Total: [6 marks]

Question 3

Question	Solution / Working	Marks	Notes
3(a)	$z_1 z_2 = (\sqrt{3}+i)(1-i) = \sqrt{3} - \sqrt{3}i + i + 1$ $= (\sqrt{3}+1) + (1-\sqrt{3})i$	M1 A1	
3(b)	$z_1 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ $z_2 = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$	A1 A1	Accept cis notation and Euler form
3(c)	$\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{2}{\sqrt{2}} = \sqrt{2}; \quad \arg = \frac{\pi}{6} - \left(-\frac{\pi}{4}\right) = \frac{5\pi}{12}$ $\frac{z_1}{z_2} = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$	M1 A1	Moduli divide, arguments subtract

Total: [6 marks]

Question 4

Question	Solution / Working	Marks	Notes
4(a)	$P(U) = P(A)P(U\lvert A) + P(B)P(U\lvert B) + P(C)P(U\lvert C)$ $= 0.5\times 0.02 + 0.3\times 0.04 + 0.2\times 0.05 = 0.01 + 0.012 + 0.01$ $= 0.032 = \frac{4}{125}$ AG	M1 A1 AG1	Tree diagram earns the M1
4(b)	$P(B\lvert U) = \frac{P(B)P(U\lvert B)}{P(U)}$ $= \frac{0.3\times 0.04}{0.032}$ $= \frac{0.012}{0.032}$ $= \frac{12}{32} = \frac{3}{8}$	M1 A1 A1 A1	M1 Bayes; A1 numerator; A1 denominator FT from (a); A1 exact fraction

Total: [7 marks]

Question 5

Question	Solution / Working	Marks	Notes
5(a)	differentiating both sides with respect to x: $2x + \left(y + x\dfrac{dy}{dx}\right) + 2y\dfrac{dy}{dx} = 0$ $(x+2y)\dfrac{dy}{dx} = -(2x+y)$ $\dfrac{dy}{dx} = -\dfrac{2x+y}{x+2y}$ AG	M1 A1 A1 AG1	M1 for implicit differentiation with the product rule on xy
5(b)	at $(1,\ 2)$: $\dfrac{dy}{dx} = -\dfrac{4}{5}$ $y - 2 = -\dfrac{4}{5}(x-1) \implies 4x + 5y = 14$	A1 A1	
5(c)	gradient of normal $= \dfrac{5}{4}$	A1	FT from (b)

Total: [7 marks]

Question 6

Question	Solution / Working	Marks	Notes
6(a)	form $\dfrac{0}{0}$: $\lim_{x\to 0}\dfrac{1-\cos 3x}{x^2} = \lim_{x\to 0}\dfrac{3\sin 3x}{2x}$ this is still $\dfrac{0}{0}$, so apply l'Hôpital again $= \lim_{x\to 0}\dfrac{9\cos 3x}{2}$ $= \dfrac{9}{2}$	M1 R1 M1 A1	R1 for recognising the repeated indeterminate form
6(b)	form $\dfrac{0}{0}$: $\lim_{x\to 0}\dfrac{10x+3}{2e^{2x}}$ $= \dfrac{0+3}{2\times 1} = \dfrac{3}{2}$	M1 A1 A1	A1 correct derivatives, A1 value

Total: [7 marks]

Question 7

Question	Solution / Working	Marks	Notes
7(a)	$(1+\tan^2 x)-\tan x = 1$ $\tan^2 x - \tan x = 0 \implies \tan x(\tan x - 1) = 0$ AG	M1 AG1	
7(b)	$\tan x = 0:\ x = 0,\ \pi,\ 2\pi$ $\tan x = 1:\ x = \frac{\pi}{4},\ \frac{5\pi}{4}$ all five solutions: $x = 0,\ \frac{\pi}{4},\ \pi,\ \frac{5\pi}{4},\ 2\pi$	A1 A1 A1 A1	A1 for each family, A1 for including both endpoints 0 and 2π, A1 all correct and no extras
7(c)	domain: $x \in \mathbb{R}$ range: $-\frac{\pi}{2} < f(x) < \frac{\pi}{2}$	A1 A1	Open interval required for the range

Total: [8 marks]

Question 8

Question	Solution / Working	Marks	Notes
8(a)	assume, for contradiction, that $\sqrt{3}$ is rational: $\sqrt{3} = \frac{p}{q}$ with $p,\ q \in \mathbb{Z},\ q \neq 0$, and p, q having no common factor $3q^2 = p^2$ so p^2 is divisible by 3; since 3 is prime, p is divisible by 3 — write $p = 3m$ $3q^2 = 9m^2 \implies q^2 = 3m^2$, so q is also divisible by 3 p and q then share the factor 3, contradicting the assumption that they are coprime; hence $\sqrt{3}$ is irrational	M1 A1 R1 A1 R1	The coprime assumption must be stated for the final R1
8(b)	try n = 11: $11^2 - 11 + 11 = 121$ $121 = 11^2$, which is not prime, so the statement fails for n = 11 — a single failing case disproves a 'for every n' claim	M1 A1 R1	The explanation of WHY it is a counterexample is required (guide: stating the example alone is not sufficient)

Total: [8 marks]

Question 9

Question	Solution / Working	Marks	Notes
9(a)	$\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ $\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	A1 A1	
9(b)	$\lvert\mathbf{u}\rvert = \sqrt{4+4+1} = \sqrt{9} = 3$ $\lvert\mathbf{v}\rvert = \sqrt{1+4+4} = 3$, so the triangle is isosceles AG	A1 AG1	
9(c)	$\mathbf{u}\cdot\mathbf{v} = (2)(1) + (2)(2) + (-1)(2) = 4$ $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}$ $= \dfrac{4}{3\times 3} = \dfrac{4}{9}$	M1 A1 M1 A1	Exact value required — no decimal
9(d)	$\mathbf{r} = (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + \lambda(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$	A1 A1	A1 position vector of A (or B), A1 direction u; any form accepted
9(e)	require (8, 5, 0) = (2 + 2λ, −1 + 2λ, 3 − λ) $x:\ 2+2\lambda = 8 \implies \lambda = 3;\quad y:\ -1+2\lambda = 5 \implies \lambda = 3;$ $z:\ 3-\lambda = 0 \implies \lambda = 3$ the same λ = 3 satisfies all three equations, so D lies on L	M1 A1 R1	All three components must be checked for the R1
9(f)	$\mathbf{u}\times\mathbf{v} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 2 & -1\\ 1 & 2 & 2\end{vmatrix}$ $= (2\cdot 2 - (-1)\cdot 2)\mathbf{i} - (2\cdot 2 - (-1)\cdot 1)\mathbf{j} + (2\cdot 2 - 2\cdot 1)\mathbf{k}$ $= 6\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$	M1 A1 A1	
9(g)	$\text{area} = \dfrac{1}{2}\lvert\mathbf{u}\times\mathbf{v}\rvert = \dfrac{1}{2}\sqrt{36+25+4}$ $= \dfrac{\sqrt{65}}{2}$	M1 A1	FT from (f). Exact surd required

Total: [18 marks]

Question 10

Question	Solution / Working	Marks	Notes
10(a)	$f'(x) = e^x + xe^x = (1+x)e^x$ $e^x > 0$ **for all** x**, so** $f'(x) = 0$ **only when** $x = -1$ AG	**M1 A1** **AG1**	M1 product rule; the reason $e^x \neq 0$ is needed for the AG
10(b)	parts with u = x, dv = e^x dx, so du = dx, v = e^x: $\int xe^x\,dx = xe^x - \int e^x\,dx$ $= xe^x - e^x + c$ $= (x-1)e^x + c$ AG	**M1** **A1** **A1** **AG1**	
10(c)	for $0 \le x \le 1$, $f(x) \ge 0$, so the area is the integral $\text{area} = \left[(x-1)e^x\right]_0^1 = (0)e^1 - (-1)e^0$ $= 0 + 1 = 1$	**M1** **A1** **A1**	Exact value 1 — a decimal from nowhere scores A0
10(d)	parts with u = x^2, dv = e^x dx: $\int x^2e^x\,dx = x^2e^x - 2\int xe^x\,dx$ using part (b): $= x^2e^x - 2(x-1)e^x + c = (x^2 - 2x + 2)e^x + c$ AG	**M1** **A1** **M1** **AG1**	Second M1 for reusing (b) — repeated integration by parts
10(e)	$V = \pi\int y^2\,dx = \pi\int_0^1 x^2e^x\,dx$ (limits 0 to 1) $= \pi\left[(x^2 - 2x + 2)e^x\right]_0^1 = \pi\left[(1)e - (2)(1)\right]$ $V = \pi(e-2)$	**M1** **A1** **A1**	$y^2 = x^2e^x$ since $\left(e^{\frac{x}{2}}\right)^2 = e^x$
10(f)	$u = x^2$ gives $du = 2x\,dx$: $\int xe^{x^2}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^{x^2} + c$	**M1** **A1**	

Total: [19 marks]

Question 11

Question	Solution / Working	Marks	Notes
11(a)	let P(n) be the statement $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ n = 1: LHS = $\cos\theta + i\sin\theta$ = RHS, so P(1) is true assume P(k) is true for some $k \in \mathbb{Z}^+$: $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ $(\cos\theta + i\sin\theta)^{k+1} = (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$ $= (\cos k\theta\cos\theta - \sin k\theta\sin\theta)$ $\quad + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$ $= \cos(k+1)\theta + i\sin(k+1)\theta$ by the compound angle identities, so P(k + 1) is true P(1) is true, and P(k) true implies P(k + 1) true; therefore by mathematical induction P(n) is true for all $n \in \mathbb{Z}^+$	**A1** **M1** **M1** **A1** **A1** **R1** **R1**	The final R1 needs the full induction conclusion, not just 'true by induction'
11(b)	$\lvert w\rvert = \sqrt{2};\quad \arg w = \frac{\pi}{4}$ $w = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$	**A1** **A1**	
11(c)	$w^8 = (\sqrt{2})^8(\cos 2\pi + i\sin 2\pi)$ $= 2^4 \times 1$ $= 16$	**M1** **A1** **A1**	
11(d)	$\arg(w^n) = \frac{n\pi}{4}$; real and negative requires $\frac{n\pi}{4} = \pi + 2k\pi$ smallest positive solution: n = 4 $w^4 = (\sqrt{2})^4(\cos\pi + i\sin\pi)$ $= -4$	**M1** **A1** **M1** **A1**	
11(e)	$w^4 = -4$ means $w = 1 + i$ is a root of $z^4 + 4 = 0$ complex roots of a real polynomial occur in conjugate pairs $z = 1+i,\ 1-i,\ -1+i,\ -1-i$	**R1** **A1 A1**	A1 for the conjugate $1 - i$; A1 for the remaining pair (the negatives, since $z^4 = (-z)^4$)

Total: [19 marks]