STADIRA
TRAINING INSTITUTE

Mathematics – Applications and Interpretation
Higher Level
Paper 1 – Marking Key

Marks: 110
2 hours

DP MOCK EXAMINATION – SET 1

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NAME	
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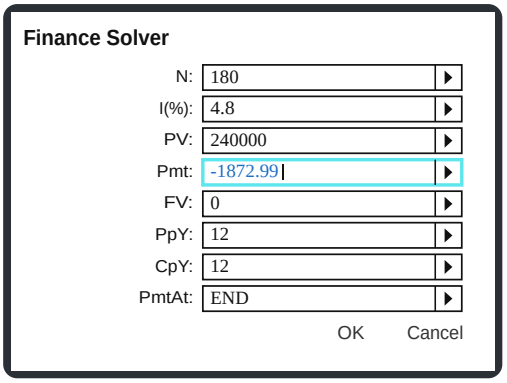
Instructions to candidates

- This marking key uses IB-style notation: M = method mark, A = accuracy mark, R = reasoning mark, AG = answer given, FT = follow through from previous part.
- Award FT marks where a candidate uses an incorrect earlier answer correctly in later working.
- Accept answers correct to three significant figures, or the accuracy specified in the question.
- GDC boxes show one efficient TI-Nspire CX II method; any valid method receives full credit.
- Screens shown are drawn recreations of the TI-Nspire CX II display.

Information

- This marking key has 11 pages.
- The number of marks for each question or part question is shown in brackets [ ].

Question 1

Question	Solution / Working	Marks	Notes
1(a)	N = 180, I = 4.8 %, PV = 240 000, FV = 0, PpY = CpY = 12 Pmt = −1872.99… monthly repayment = 1870 shillings (1872.99) **TI-Nspire:** menu → 8: Finance → 1: Finance Solver… 	**M1** **A1** **A1**	
1(b)	1872.9946… × 180 − 240 000 = 337 139.04 − 240 000 = 97 100 shillings (97 139.04)	**M1** **A1**	FT from (a)

Total: [5 marks]

Question 2

Question	Solution / Working	Marks	Notes
2(a)	$24 = 2^3 \times 3$ $\log_{10} 24 = 3\log_{10} 2 + \log_{10} 3$ = 3p + q	**M1** **M1** **A1**	M1 correct factorisation; M1 log of a product and of a power
2(b)	2 ln x = ln x² and ln 9 + ln 4 = ln 36 x² = 36 x = 6 (reject x = −6, since ln x requires x > 0)	**M1** **A1** **A1**	A1 for rejecting the negative root

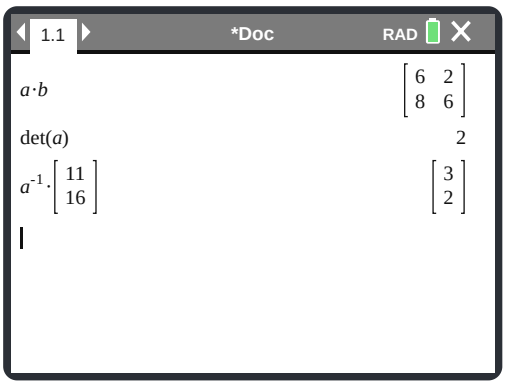
Total: [6 marks]

Question 3

Question	Solution / Working	Marks	Notes
3(a)	$2.4 \times 0.62^5 = 0.220$ m (0.219872)	**A1**	
3(b)	after the first drop of 2.4 m, each bounce contributes an up and a down journey $\text{total} = 2.4 + 2 \times \dfrac{2.4 \times 0.62}{1 - 0.62}$ $= 2.4 + 2 \times \dfrac{1.488}{0.38} = 2.4 + 7.8316$ = 10.2 m (10.2316)	**M1** **M1** **A1** **A1**	M1 for the factor of 2; M1 for $\dfrac{a}{1-r}$ with a = 1.488
3(c)	$\lvert r \rvert < 1$	**A1**	

Total: [6 marks]

Question 4

Question	Solution / Working	Marks	Notes
4(a)	$AB = \begin{pmatrix} 6 & 2 \\ 8 & 6 \end{pmatrix}$	**M1 A1**	M1 for correct row-by-column multiplication
4(b)	det A = 3(2) − 1(4) = 2	**A1**	
4(c)	$A^{-1} = \dfrac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$ $= \begin{pmatrix} 1 & -0.5 \\ -2 & 1.5 \end{pmatrix}$	**M1** **A1**	FT from (b)
4(d)	the system is $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 16 \end{pmatrix}$, so $\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 11 \\ 16 \end{pmatrix}$ x = 3, y = 2 **TI-Nspire:** Store the matrices with the template on a Calculator page; `det(a)` and `a^(−1)` work directly. 	**M1** **A1**	

Total: [7 marks]

Question 5

Question	Solution / Working	Marks	Notes
5(a)	$(3 + 4i)(1 - 2i) = 3 - 6i + 4i - 8i^2$ $= 3 + 8 - 2i = 11 - 2i$	**M1** **A1**	Uses $i^2 = -1$
5(b)	$\lvert z_1 \rvert = \sqrt{3^2 + 4^2} = 5$ $\arg(z_1) = \arctan\left(\frac{4}{3}\right)$ = 0.927 radians (0.927295, i.e. 53.1°)	**A1** **M1** **A1**	z_1 is in the first quadrant, so no adjustment is needed
5(c)	$z_1 = 5e^{0.927i}$ (equivalently 5(cos 0.927 + i sin 0.927))	**A1** **A1**	FT from (b)

Total: [7 marks]

Question 6

Question	Solution / Working	Marks	Notes
6(a)	$g(3) = 3^2 + 1 = 10$ $f(10) = 2(10) - 5 = 15$	**M1** **A1**	
6(b)	$(g \circ f)(x) = (2x - 5)^2 + 1$ $= 4x^2 - 20x + 26$	**M1** **A1**	
6(c)	let y = 2x − 5, so $x = \frac{y + 5}{2}$ $f^{-1}(x) = \frac{x + 5}{2}$	**M1** **A1**	
6(d)	$f^{-1}(7) = 6$ and $f(6) = 2(6) - 5 = 7$ AG	**AG1**	

Total: [7 marks]

Question 7

Question	Solution / Working	Marks	Notes
7(a)	$\text{arc} = r\theta = 9 \times \dfrac{2\pi}{5}$ = 11.3 cm (11.3097)	M1 A1	Radian mode
7(b)	$\text{area} = \dfrac{1}{2}r^2\theta = \dfrac{1}{2} \times 81 \times \dfrac{2\pi}{5}$ = 50.9 cm² (50.8938)	M1 A1	
7(c)	$\text{area} = \dfrac{1}{2} \times 81 \times \sin\left(\dfrac{2\pi}{5}\right) = 38.5 \text{ cm}^2$ (38.5178)	A1	
7(d)	segment = sector − triangle = 50.8938 − 38.5178 = 12.4 cm² (12.3760)	M1 A1	FT from (b), (c)

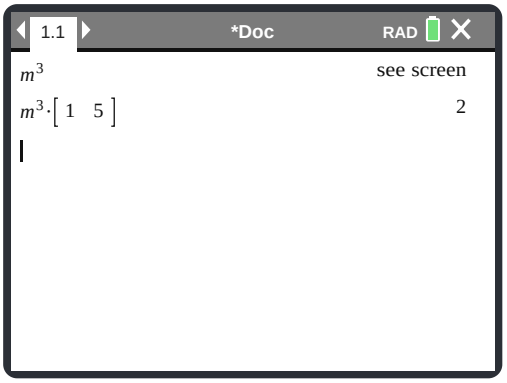
Total: [7 marks]

Question 8

Question	Solution / Working	Marks	Notes
8(a)	$\lvert\mathbf{a}\rvert = \sqrt{3^2 + (-1)^2 + 2^2} = \sqrt{14}$ = 3.74 (3.74166)	M1 A1	
8(b)	a·b = 3(1) + (−1)(4) + 2(−2) = 3 − 4 − 4 = −5	M1 A1	
8(c)	$\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert} = \dfrac{-5}{\sqrt{14}\sqrt{21}}$ cos θ = −0.291606… θ = 107° (106.954°, or 1.87 radians)	M1 A1 A1	FT from (a), (b). The negative dot product gives an obtuse angle

Total: [7 marks]

Question 9

Question	Solution / Working	Marks	Notes
9(a)	A: 2, B: 3, C: 3, D: 2, E: 2	**A1 A1**	A1 for at least three correct; A1 for all five
9(b)	exactly two vertices (B and C) have odd degree so there is no Eulerian circuit, but an Eulerian trail exists, starting at B and finishing at C (or the reverse)	**R1** **A1**	
9(c)	the number of walks of length 3 from A to E is the entry in row A, column E of M^3 M^3 has entry 2 in that position so there are 2 such walks **TI-Nspire:** Store the adjacency matrix as m, then evaluate `m^3` and read the entry in row 1, column 5. 	**R1** **M1** **A1**	

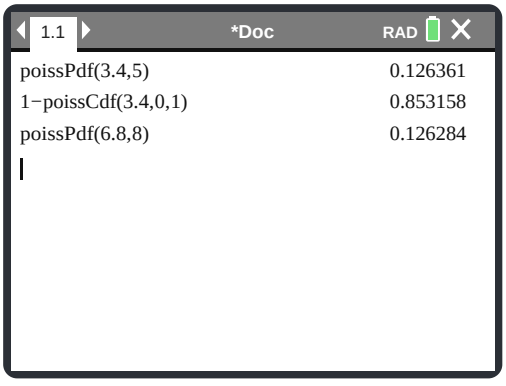
Total: [7 marks]

Question 10

Question	Solution / Working	Marks	Notes
10(a)	sorted by weight: BC 3, AC 4, DE 5, AB 6, CD 7, BD 8, CE 9 choose BC (3), then AC (4), then DE (5) reject AB (6) because it would create the cycle A–C–B–A choose CD (7); the tree now has 4 edges and connects all five vertices, so the algorithm stops	**M1** **A1** **A1** **A1**	M1 edges considered in increasing order; the rejection of AB must be stated
10(b)	3 + 4 + 5 + 7 = 19, that is 19 thousand dollars	**A1**	FT from (a)
10(c)	a spanning tree must connect all n vertices, and a tree contains no cycles starting from one vertex, each additional edge can connect exactly one new vertex, so n − 1 edges are needed and any further edge would create a cycle	**R1** **R1**	

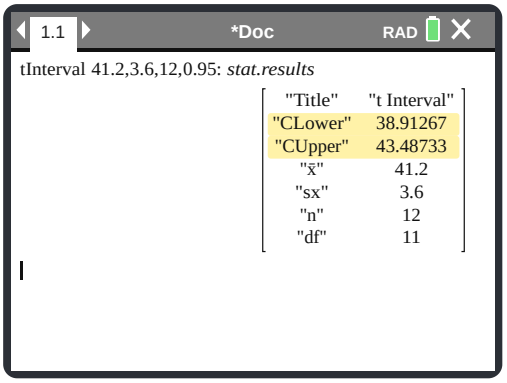
Total: [7 marks]

Question 11

Question	Solution / Working	Marks	Notes
11(a)	the calls arrive independently of one another (or: at a uniform average rate over the interval)	**A1**	
11(b)	X ~ Po(3.4), P(X = 5) = 0.126 (0.126361…) **TI-Nspire:** menu → 6: Statistics → 5: Distributions → J: Poisson Pdf, and K: Poisson Cdf for (c). 	**M1 A1**	
11(c)	P(X ≥ 2) = 1 − P(X ≤ 1) = 0.853 (0.853158…)	**M1** **A1**	
11(d)	for 20 minutes the mean is 2 × 3.4 = 6.8 P(Y = 8) = 0.126 (0.126284…)	**M1** **A1**	M1 for scaling the mean to the new interval

Total: [7 marks]

Question 12

Question	Solution / Working	Marks	Notes
12(a)	the t-distribution, because the population standard deviation σ is unknown and is estimated from the sample	**R1**	
12(b)	df = 11, so t* = 2.20099 $\text{margin} = 2.20099 \times \frac{3.6}{\sqrt{12}} = 2.28733$ 41.2 ± 2.28733 confidence interval (38.9, 43.5) (38.9127, 43.4873) **TI-Nspire:** menu → 6: Statistics → 6: Confidence Intervals → 2: t Interval, choose Stats, x̄ = 41.2, sx = 3.6, n = 12. 	**A1** **M1** **A1** **A1**	
12(c)	we are 95 % confident that the true mean lifetime of all such batteries lies between 38.9 and 43.5 hours that is, if many samples of 12 were taken, about 95 % of the intervals constructed this way would contain the population mean	**A1** **R1**	Do not accept 'there is a 95 % probability the mean is in this interval'

Total: [7 marks]

Question 13

Question	Solution / Working	Marks	Notes
13(a)	$E(3X - 5) = 3E(X) - 5 = 3(24) - 5$ $= 67$	**M1** **A1**	
13(b)	$\text{Var}(3X - 5) = 3^2\text{Var}(X) = 9 \times 9$ $= 81$	**M1** **A1**	The constant −5 does not affect the variance
13(c)	$E(X_1 + X_2 + X_3) = 3 \times 24 = 72$ since the variables are independent, $\text{Var}(X_1 + X_2 + X_3) = 3 \times 9$ $= 27$ (note this is not the same as $\text{Var}(3X) = 81$)	**A1** **M1** **A1**	

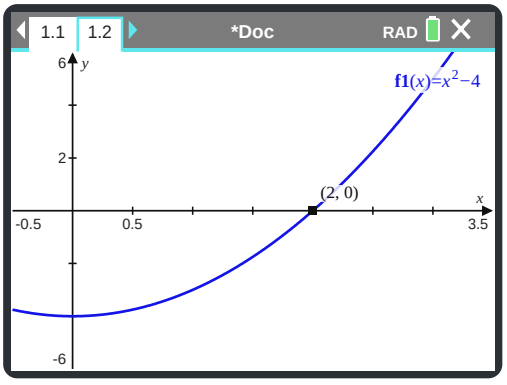
Total: [7 marks]

Question 14

Question	Solution / Working	Marks	Notes
14(a)	product rule with $u = x^2$ and $v = e^{3x}$ $\frac{dy}{dx} = 2xe^{3x} + 3x^2e^{3x}$ $= xe^{3x}(2 + 3x)$	**M1** **A1** **A1**	M1 product rule; A1 each correct term
14(b)	at $x = 1$: $1 \times e^3 \times 5 = 5e^3$ $= 100$ (100.428)	**M1** **A1**	FT from (a)
14(c)	chain rule with the inner function $x^2 + 1$ $\frac{dy}{dx} = \frac{2x}{x^2 + 1}$	**M1** **A1**	

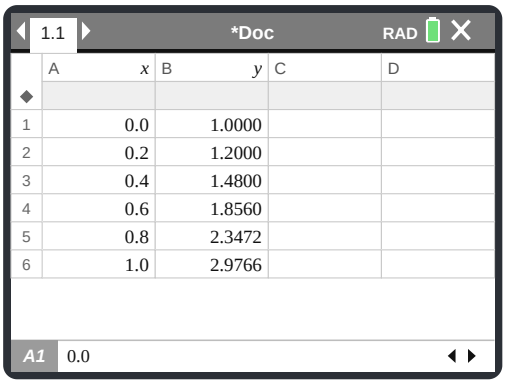
Total: [7 marks]

Question 15

Question	Solution / Working	Marks	Notes
15(a)	$\int \sin(3x)\,dx = -\frac{1}{3}\cos(3x) + c$	**A1 A1**	A1 for the cosine; A1 for the factor $-\frac{1}{3}$ and + c
15(b)	u = x² + 4 gives du = 2x dx, so 6x dx = 3 du $\int 3u^2\,du = u^3 + c$ $= (x^2+4)^3 + c$	**M1** **A1** **A1**	M1 substitution carried through including du
15(c)	the curve is below the x-axis for 0 ≤ x < 2 and above it for 2 < x ≤ 3, so the integral must be split at x = 2 $A = \int_0^2 (4 - x^2)\,dx + \int_2^3 (x^2 - 4)\,dx$ $= \frac{16}{3} + \frac{7}{3} = \frac{23}{3} = 7.67$ 	**R1** **M1** **A1**	A candidate who integrates straight from 0 to 3 gets −3 (3 after taking the absolute value) and scores R0 M0 A0

Total: [8 marks]

Question 16

Question	Solution / Working	Marks	Notes
16(a)	$y_{n+1} = y_n + h \times f(x_n, y_n) = y_n + h(x_n + y_n)$	**A1**	
16(b)	x = 0, 0.2, 0.4, 0.6, 0.8, 1.0 y = 1, 1.2, 1.48, 1.856, 2.3472, 2.97664 **TI-Nspire:** Lists & Spreadsheet: put 0, 0.2, … in column A, then in B2 enter `=b1+0.2*(a1+b1)` and fill down. 	**M1** **A1 A1 A1**	M1 recursion applied; A1 for y(0.2) and y(0.4); A1 for y(0.6) and y(0.8); A1 for y(1)
16(c)	y(1) ≈ 2.98 (2.97664)	**A1**	FT from (b)
16(d)	exact value = 2e − 1 − 1 = 2e − 2 = 3.43656 $\frac{\lvert 2.97664 - 3.43656 \rvert}{3.43656} \times 100 = 13.4\,\%$ (13.3832)	**M1** **A1**	Euler's method underestimates here because the solution curve is concave up

Total: [8 marks]