STADIRA
TRAINING INSTITUTE

Mathematics – Analysis and Approaches
Standard Level
Paper 1
Marks: 80
1 hour 30 minutes

DP MOCK EXAMINATION – SET 1

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Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions. Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly.
- A clean copy of the mathematics: analysis and approaches formula booklet is required for this paper.
- The maximum mark for this examination paper is [80 marks].

Information

- This question paper has 11 pages.
- The number of marks for each question or part question is shown in brackets [ ].

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

No calculator is allowed for this paper. All answers must be given exactly, for example as fractions, surds or multiples of π. Decimal approximations will not receive full credit.

Section A

[30 marks] *Answer all questions.*

1. [Maximum mark: 6]

(a) Simplify $\frac{(3x^2y^3)^3}{9x^4y^2}$, giving your answer in the form ax^my^n. [3]

(b) Prove that the sum of any three consecutive integers is always divisible by 3. [3]

2. [Maximum mark: 6]

Consider the binomial expansion of $(2+3x)^5$.

(a) Find the first three terms of the expansion, in ascending powers of x. [3]

(b) Find the coefficient of x^3 in the expansion. [3]

3. [Maximum mark: 6]

The quadratic equation $k x^2 + (k+2)x + 3 = 0$, where k ≠ 0, has two equal real roots.

(a) Show that $\Delta = k^2 - 8k + 4$. [3]

(b) Hence find the exact values of k. [3]

4. [Maximum mark: 6]

Let θ be an acute angle with $\sin\ \theta = \frac{3}{5}$.

(a) Find the exact value of cos θ, and hence find the exact value of sin 2θ. [4]

(b) Find the exact value of cos 2θ. [2]

5. [Maximum mark: 6]

Consider the curve $y = x^3 - 4x$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the tangent to the curve at the point where x = 2, giving your answer in the form y = mx + c. [4]

Section B

[50 marks] *Answer all questions.*

6. [Maximum mark: 16]

Consider the function $f(x) = x^2 - 6x + 11$, defined for $x \in \mathbb{R}$.

(a) Write f(x) in the form $(x - h)^2 + k$, where h and k are integers. [3]

(b) Write down the coordinates of the vertex of the graph of f, and the equation of its axis of symmetry. [2]

(c) Write down the range of f. [1]

(d) Explain why f does not have an inverse function when its domain is $\mathbb{R}$, and state the largest value of a for which f has an inverse on the domain $x \geq a$. [3]

The domain of f is now restricted to $x \geq 3$.

(e) Find an expression for $f^{-1}(x)$. [4]

(f) Write down the domain of f^{-1}. [1]

The graph of f, with domain $\mathbb{R}$, is translated 2 units in the positive x-direction and 5 units in the negative y-direction to give the graph of a function g.

(g) Find an expression for g(x), giving your answer in the form $x^2 + bx + c$. [2]

7. [Maximum mark: 17]

Consider the function $f(x) = x^3 - 3x^2$.

(a) Find $f'(x)$, and hence find the coordinates of the two stationary points on the graph of f. [5]

(b) Find $f''(x)$, and use it to determine the nature of each stationary point. [3]

(c) Find the coordinates of the points where the graph of f meets the x-axis. [2]

(d) Sketch the graph of f for $-1 \le x \le 4$, showing the stationary points and the x-intercepts. [3]

(e) Find the exact area of the region enclosed by the graph of f and the x-axis between x = 0 and x = 3. [4]

8. [Maximum mark: 17]

Consider the function $f(x) = 3\ \sin(2x) + 1$, for $0 \le x \le 2\pi$.

(a) Write down the amplitude, the period, and the range of f. [3]

(b) Solve the equation $f(x) = \frac{5}{2}$ for $0 \le x \le 2\pi$, giving your answers as exact multiples of π. [6]

(c) Sketch the graph of f for $0 \le x \le 2\pi$, showing clearly the coordinates of the maximum and minimum points. [4]

(d) State the number of solutions of the equation f(x) = 4 for $0 \le x \le 2\pi$, and justify your answer. [4]