Geometry & Trigonometry · Unit G06 · Foundations

Circles and Compound Shapes

Foundations · MYP 1–3 · 60 practice questions · Criteria A, B, C, D

The sixth unit of the Geometry & Trigonometry strand, and the partner of G05. G05 covered every plane figure with straight edges. This one adds the curve: circles, halves and quarters of them, and the compound shapes and shaded regions they build.

A tank base, a washer, a shade sail with a rounded end, a centre-pivot irrigation circle: the moment a shape stops having straight edges, one formula does all the work — **$A = \pi r^2$**. And the moment a shape is more than one piece, one habit does the rest: **split it, work out each piece, then add or subtract**. This unit is those two ideas and the discipline that keeps them honest: halve the diameter before you square it, and keep π in the working until the very last line.

By the end of this unit you can

• **Calculate** the area of a circle from its radius, and from its diameter, giving the answer exactly in terms of π and to 3 significant figures

• **Calculate** the area of a semicircle and of a quarter circle, and **describe** which fraction of the whole circle each one is

• **Calculate** the area of a compound shape by splitting it into pieces and adding them, without counting any region twice

• **Calculate** a shaded area by the hole method, including the ring between two concentric circles

• **Determine** what fraction of a shape is shaded, and **describe** why that fraction does not always depend on the size

• Run $A = \pi r^2$ backwards to a missing radius or diameter, and **describe** why rounding early loses the third significant figure

The one idea behind all of it

A circle’s area is **the radius squared, multiplied by π** — and π is not a decimal you look up, it is the number that says how many radius-squares

fit inside the circle. It is a little more than three of them. Every other shape in this unit is built from circles and straight-edged pieces: a semicircle is half a circle, a quarter circle is a quarter of one, a compound shape is pieces **added**, and a shaded region is a whole with a hole **subtracted**. Nothing new is needed for any of them. What is needed is care: the formula wants the **radius**, never the diameter, and it wants it **squared**, never doubled.

A = πr² — halved for a semicircle, quartered for a quarter circle, added for a compound, subtracted for a hole. Two habits do most of the work: write the radius down first, every time, even when the question gave you the diameter; and list the pieces before you calculate, marking each one + or −.

Every answer carries its unit, squared: cm², m² or mm². A bare 78.5 is not an answer. Give the exact value in terms of π where you can, and anything that is not exact to 3 significant figures. Round, never truncate, and round only once, at the very end.

Where this unit sits

This unit finishes what G05 Cover the Surface started. G05 covered every plane figure with straight edges — rectangle, triangle, parallelogram, rhombus, kite and trapezium — and those formulas are assumed here, not retaught. What is new is the curve. Every compound shape in this unit is one of those straight-edged shapes with a circle, a semicircle or a quarter circle added to it or taken out of it.

Area, not perimeter. G04 *Around the Edge* did the way round, including the circumference C = πd. This unit does the space inside. The two formulas look alike and are not: C = πd uses the diameter and gives a length in cm; A = πr² uses the radius and gives an area in cm². If an answer needs a squared unit, it is this unit's question.

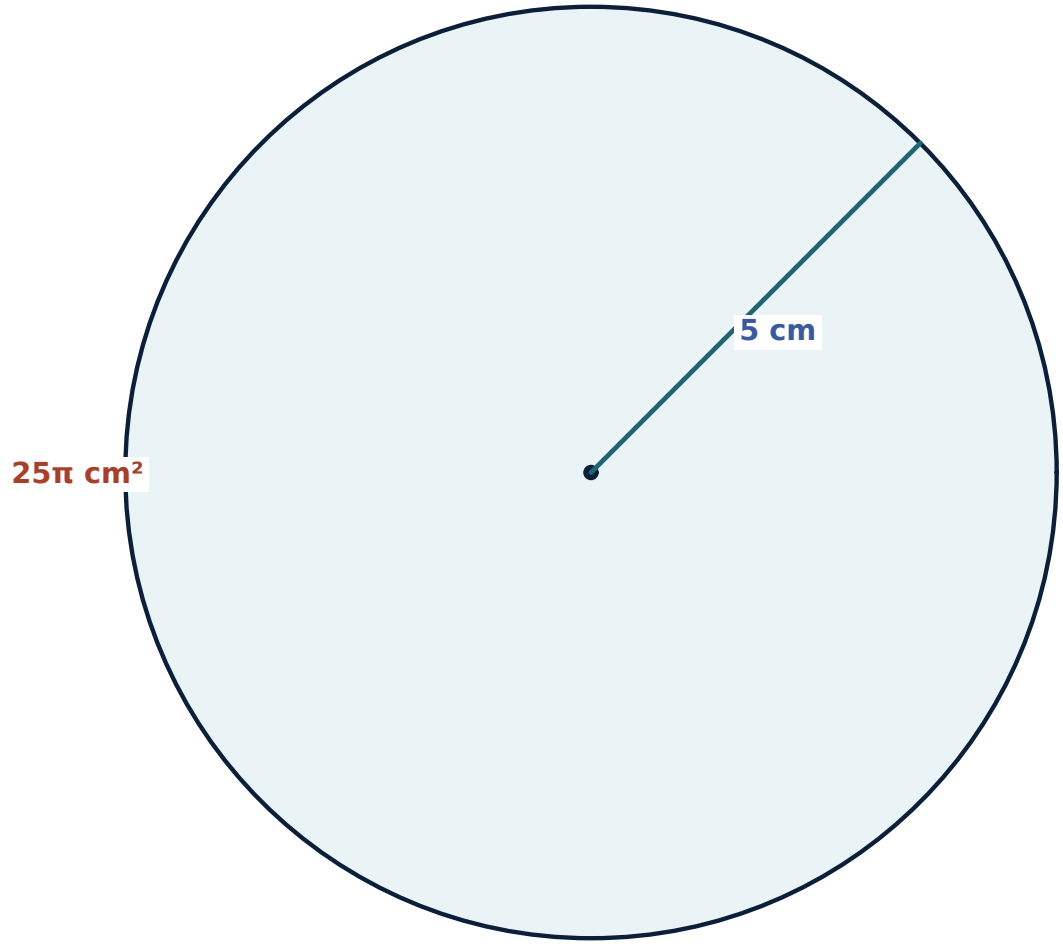
Whole circles, halves and quarters only. A slice cut at any other angle is a *sector*, and sectors, arcs and segments are G13 *Slices of the Circle*. Curved surfaces of solids are G08 *Wrap It Up*. **No new number work:** squaring is Number Unit N10 *Squares, Roots & Powers* and rounding is N04 *Round It Right*. **House rules:** every answer carries its unit, squared — a bare "78.5" is not an answer — and anything that is not exact is given to 3 significant figures, rounded, never truncated.

Concept 1 · The circle: A = πr²

Square the radius, then multiply by π — and the radius is half the diameter

A circle’s area is **π × the radius squared**. Nothing else about the circle is needed, and nothing else will do: not the diameter, not the distance round. If a question hands you the diameter, **halve it first** and write the radius down on its own line — because squaring the diameter by mistake makes the answer four times too big, and it is the single commonest error in the topic. Leave the answer as a multiple of π if you can; round it only when you must.

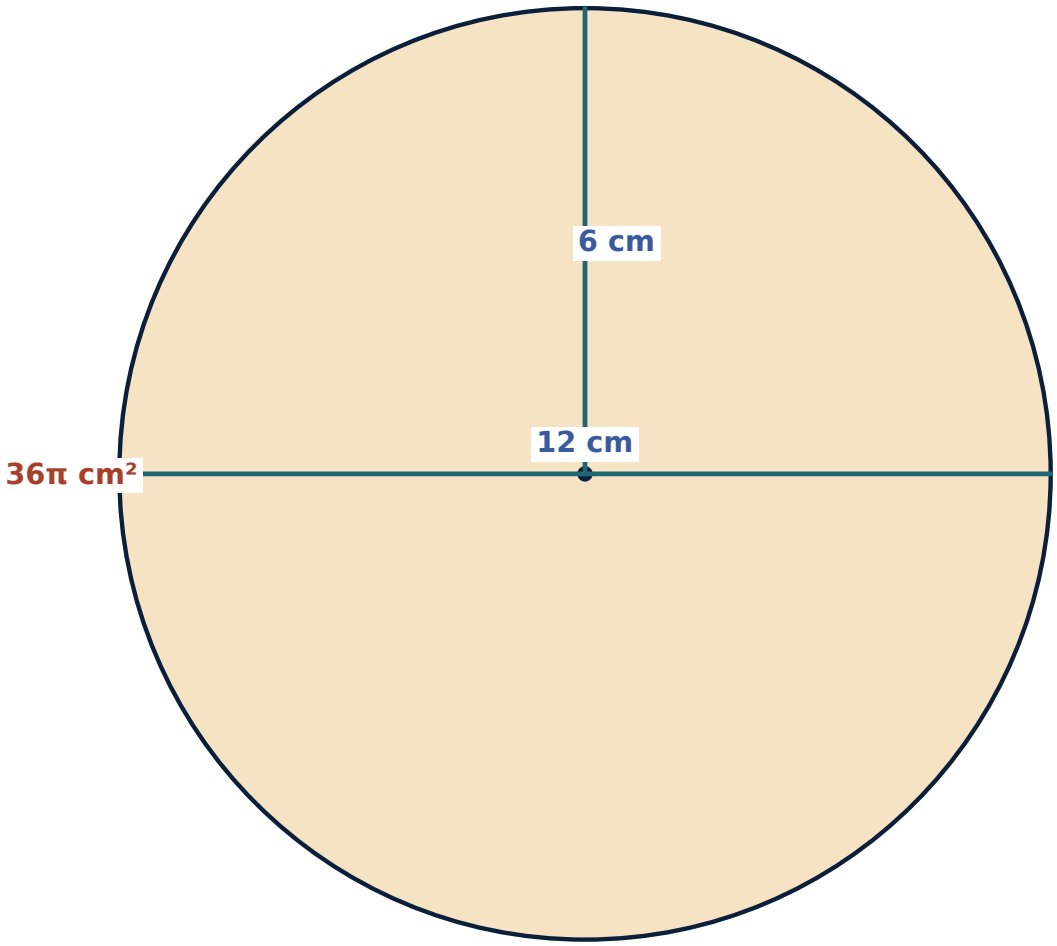
One circle, one radius, one area



Every point on the rim is the same distance from the centre.

The radius is the only measurement a circle needs. Square it, multiply by π, and you have the space inside — exactly, in terms of π, before any rounding happens at all.

When the diameter is what you are given



Halve the diameter first. Then square it.

The 12 cm across the middle is the diameter, and the formula does not want it. Halve it to 6 cm, then square that. Squaring the 12 would give four times too much.

Worked example 1 · Tank base, from the diameter

1	A circular tank has a base of diameter 12 m. Which measurement does the formula want?	**the radius, not the diameter**
2	Halve the diameter first.	**r = 12 ÷ 2 = 6 m**
3	Write the formula.	**A = πr²**
4	Substitute, and square the radius.	**A = π × 6² = π × 36**
5	Give the exact answer first.	**36π m²**
6	Now round it, once, to 3 significant figures.	**113 m²**

Squaring the 12 instead of the 6 would give 144π m², four times too much. The error is always a factor of four, which makes it easy to spot — and easy to make.

Shape	Formula	What the letters mean	The commonest slip
Circle, from the radius	A = πr²	r is the radius	using the diameter in place of r
Circle, from the diameter	A = π × (d ÷ 2)²	d is the diameter	squaring d before halving it
Semicircle	A = ½ × πr²	half a whole circle	forgetting the half
Quarter circle	A = ¼ × πr²	a quarter of a whole circle	halving instead of quartering
Compound, by addition	A = piece + piece + …	the pieces must not overlap	counting one region twice
Compound, by subtraction	A = whole − hole	the hole sits entirely inside the whole	adding when you should subtract

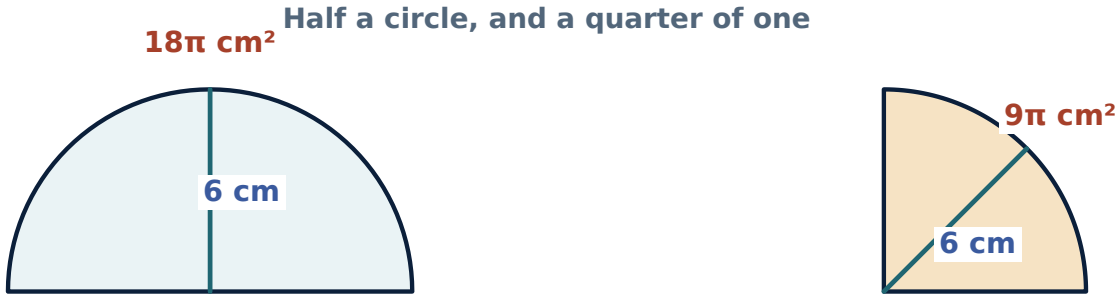
Radius	Area exactly	Area to 3 s.f.
3 cm	9π cm²	28.3 cm²
4 cm	16π cm²	50.3 cm²
5 cm	25π cm²	78.5 cm²
6 cm	36π cm²	113 cm²
10 cm	100π cm²	314 cm²

EXAMPLE A tank base is 12 m across. That is the diameter, so the radius is 6 m and the area is $\pi \times 6^2 = \mathbf{36\pi\ m^2}$, which is **113 m²** to 3 significant figures. Squaring the 12 would give 452 m² — larger than the 12 m by 12 m square the circle fits inside, which is how you know at a glance that it is wrong.

Concept 2 · Half a circle, and a quarter of one

Work out the whole circle first, then take your fraction of it

A semicircle is **half** a circle and a quarter circle is a **quarter** of one, so the safe method is always the same: work out the **whole circle first**, then take your fraction of it. Halving the radius is not halving the circle — it leaves you with a quarter of the area, because the radius is squared. A quarter circle is sometimes called a **quadrant**, and its two straight edges are radii.



Both pieces come from the same circle of radius 6 cm, whose area is 36π cm². Half of that is 18π cm² and a quarter of it is 9π cm². Work out the whole circle first, then take your fraction of it.

Worked example 2 · Half a circle, and a quarter of one

1	A semicircular window has a diameter of 14 cm. Find the radius.	**r = 7 cm**
2	Work out the whole circle it came from.	**49π cm²**
3	A semicircle is half of that.	**½ × 49π = 24.5π cm²**
4	Round, once, at the end.	**77.0 cm²**
5	A quarter circle of the same radius would be a quarter of the whole.	**12.25π cm²**
6	Check: does the quarter come to half the semicircle?	**yes — 12.25π is half of 24.5π**

Always work out the whole circle first and then take your fraction of it. Trying to halve the radius instead of the area is the mistake, and it gives a quarter of the right answer.

Piece	Fraction	Formula	The piece when r = 6 cm
Whole circle	1	$A = \pi r^2$	36π cm²
Semicircle	½	$A = \frac{1}{2}\pi r^2$	18π cm²
Quarter circle	¼	$A = \frac{1}{4}\pi r^2$	9π cm²

EXAMPLE A semicircular window is 14 cm across. The radius is 7 cm, so the whole circle would be 49π cm² and the window is half of that: **24.5π cm²**, or **77.0 cm²** to 3 significant figures. Note the trailing zero — “77” would claim only two significant figures.

Concept 3 · Compound shapes: add the pieces

Split the shape into pieces you already know, and make sure they do not overlap

A **compound shape** is one shape made of several you already know. Split it, work out each piece, and add. Two things make this go wrong: pieces that **overlap**, so a region is counted twice, and pieces that are **invented**, so something is counted that is not there. Write the list of pieces down before any arithmetic. And watch for the shortcut: two semicircles of the same radius are **one whole circle**.

A compound shape: add the pieces up



A rectangle with a semicircular end on each side. The two ends have the same radius, so together they are one whole circle — which is the quickest way to add them. Nothing overlaps, so nothing is counted twice.

Worked example 3 · A rectangle with two semicircular ends

1	A table top is a rectangle 20 cm by 10 cm with a semicircular end on each of the short sides. List the pieces.	**one rectangle, two semicircles**
2	The semicircles sit on the 10 cm sides, so what is their radius?	**r = 10 ÷ 2 = 5 cm**
3	Work out the rectangle.	**20 × 10 = 200 cm²**
4	The two semicircles have the same radius, so together they are one whole circle.	**25π cm²**
5	Add the pieces. Keep π for now.	**200 + 25π cm²**
6	Round, once, at the end.	**279 cm²**

Two semicircles of the same radius are one circle, which saves a line of working. What must never happen is counting them as two whole circles: that is 357 cm² and it is wrong.

The shape	How it splits	The working	The area
A rectangle 20 cm by 10 cm with a semicircular end on each side	rectangle + two semicircles, which are one whole circle of radius 5 cm	200 + 25π	279 cm²
A square of side 10 cm with a semicircle of diameter 10 cm on top	square + semicircle of radius 5 cm	100 + 12.5π	139 cm²
The ring between circles of radius 7 cm and 3 cm	big circle − small circle	49π − 9π	126 cm²
A square of side 10 cm with its inscribed circle removed	square − circle of radius 5 cm	100 − 25π	21.5 cm²

EXAMPLE A table top is a rectangle 20 cm by 10 cm with a semicircular end on each 10 cm side. The rectangle is 200 cm². The two ends are semicircles of radius 5 cm, so together they are one circle: 25π cm². The total is 200 + 25π = **279 cm²**. Counting each end as a whole circle gives

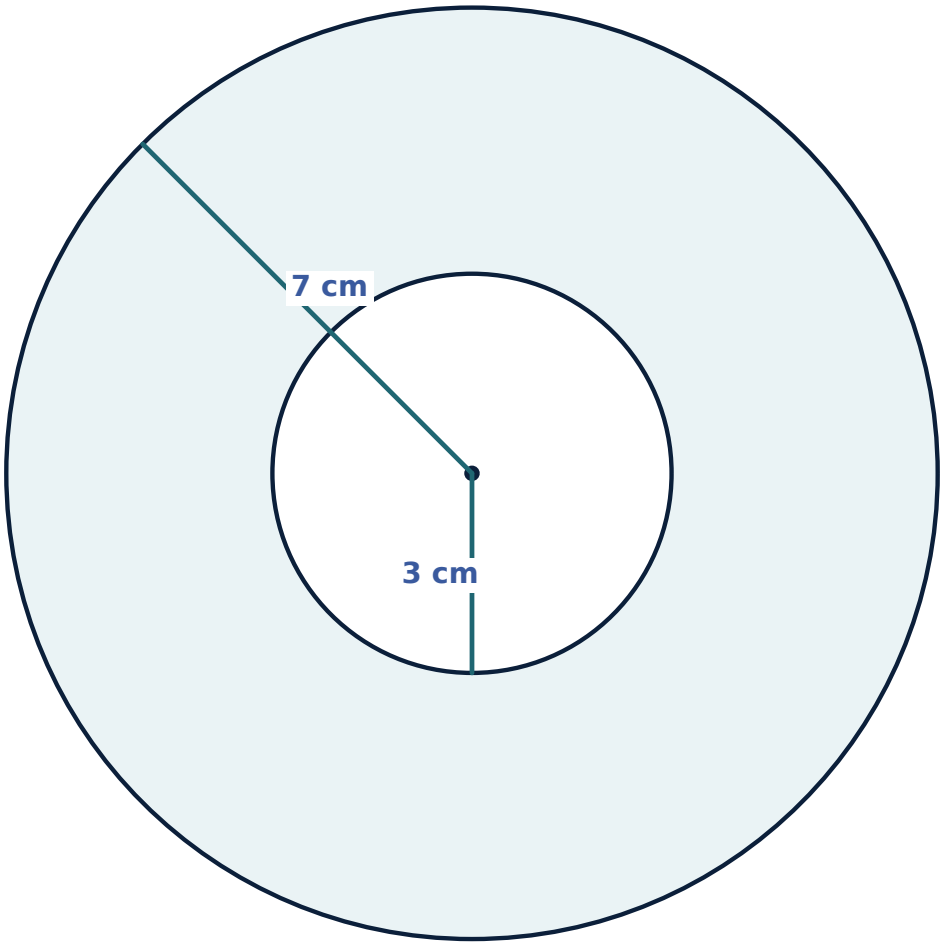
357 cm², and it is wrong.

Concept 4 · Compound shapes: the hole method

Whole minus hole — concentric circles, washers and rings

When a shape has a piece **missing**, do not try to measure what is left directly. Work out the **whole**, work out the **hole**, and subtract. Circles that share a centre are called **concentric**, and the ring between two of them — a washer, a path round a pond, a running track — is an **annulus**. Do the subtraction while π is still there: 49π − 9π = 40π is exact and instant.

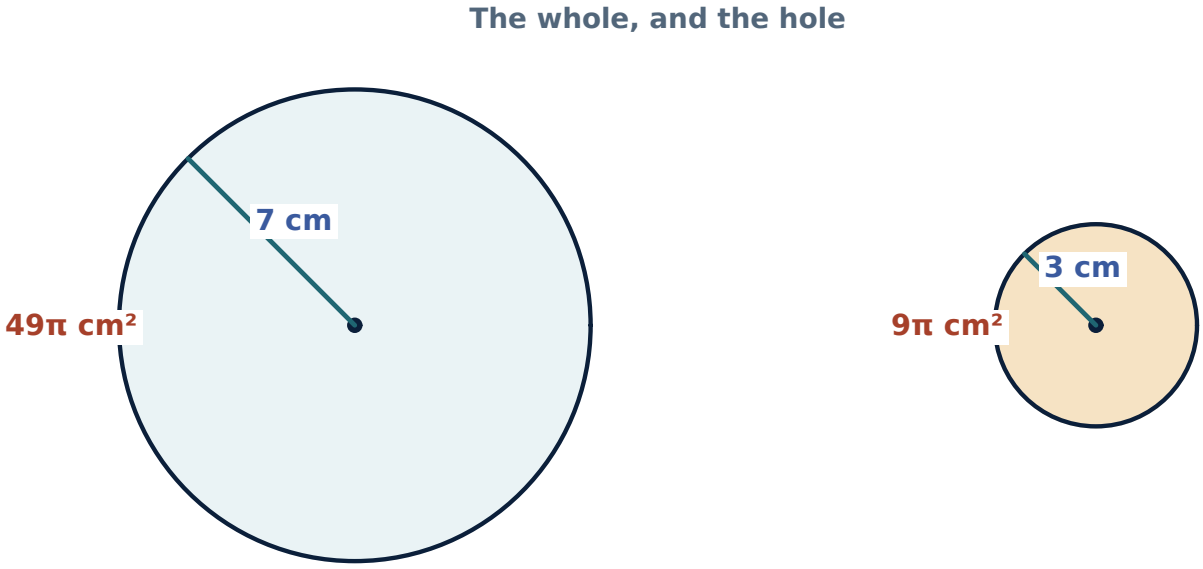
A washer: the ring between two circles



Two circles, one centre: the shaded ring is an annulus.

Circles that share a centre are called **concentric***, and the ring between them is an* **annulus***. The shaded metal is the big circle with the small one taken away — the hole*

method, in one step.



49π cm² − 9π cm² = 40π cm², which is 126 cm²

The same washer, drawn as two panels. Work out the whole circle, work out the hole, and subtract. Keeping π in both answers until the very last line is what protects the third significant figure.

Worked example 4 · The washer: whole minus hole

1	A metal washer has an outer radius of 7 cm and a hole of radius 3 cm. What are the two circles called?	**concentric — they share a centre**
2	Work out the whole disc.	**π × 7² = 49π cm²**
3	Work out the hole.	**π × 3² = 9π cm²**
4	Subtract, keeping π.	**49π − 9π = 40π cm²**
5	Round, once, at the end.	**126 cm²**
6	Check the size: the ring must be less than the whole disc.	**126 cm² is less than 154 cm²**

Subtracting in terms of π is the whole trick: 49π − 9π = 40π is exact, while 154 − 28.3 is two roundings already.

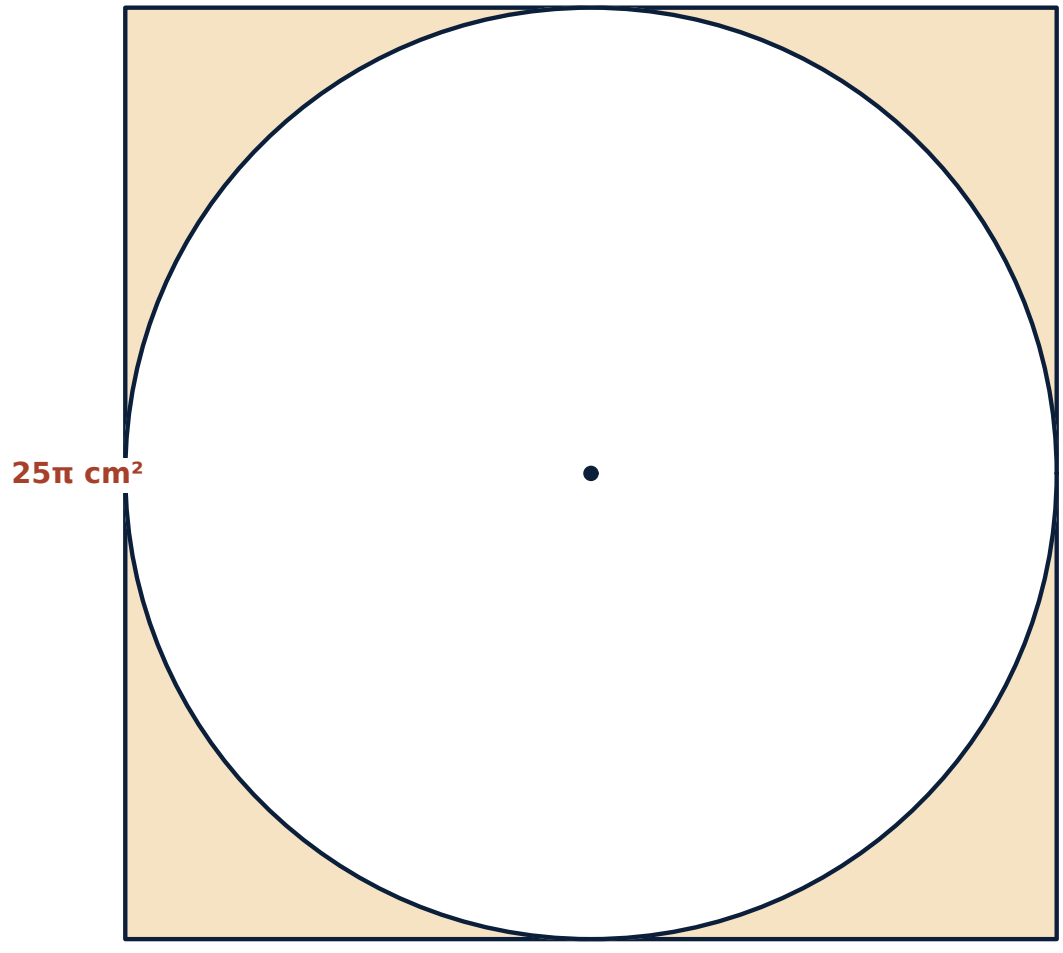
EXAMPLE A washer has an outer radius of 7 cm and a hole of radius 3 cm. The whole disc is 49π cm² and the hole is 9π cm², so the metal is 49π − 9π = **40π cm²**, which is **126 cm²**. The whole subtraction happened before any rounding did.

Concept 5 · Shaded regions, and the fraction that is shaded

The shading is what is left, and the fraction is often the same whatever the size

A **shaded region** question is the hole method with the shading telling you which part is wanted. Read the picture: the shaded part is almost always **whole − hole**. Then there is a second, better question: what **fraction** of the whole is shaded? For a circle drawn inside a square, touching all four sides, that fraction is **1 − π÷4** — about a fifth — and it is the same for every square, whatever its size, because the side length cancels.

A shaded region: square minus its inscribed circle



The four corners are what is left.

100 cm²

The circle just touches all four sides, so its diameter is the side of the square and its radius is half of that. The shaded corners are 100 − 25π cm², which is 21.5 cm² — and that is 21.5% of the square, whatever the square's size.

Worked example 5 · A square with its circle cut out

1	A circle is drawn inside a square of side 10 cm so that it touches all four sides. What is the circle’s diameter?	**10 cm, the side of the square**
2	So what is its radius?	**r = 5 cm**
3	Work out the square.	**100 cm²**
4	Work out the circle.	**25π cm²**
5	The shaded part is what is left. Subtract, then round once.	**100 − 25π = 21.5 cm²**
6	What fraction of the square is shaded?	**1 − π÷4, which is 21.5%**

That fraction is the same for **every** square with its circle inscribed, whatever the size — the 10s cancel. A square always keeps about a fifth of itself outside its own circle.

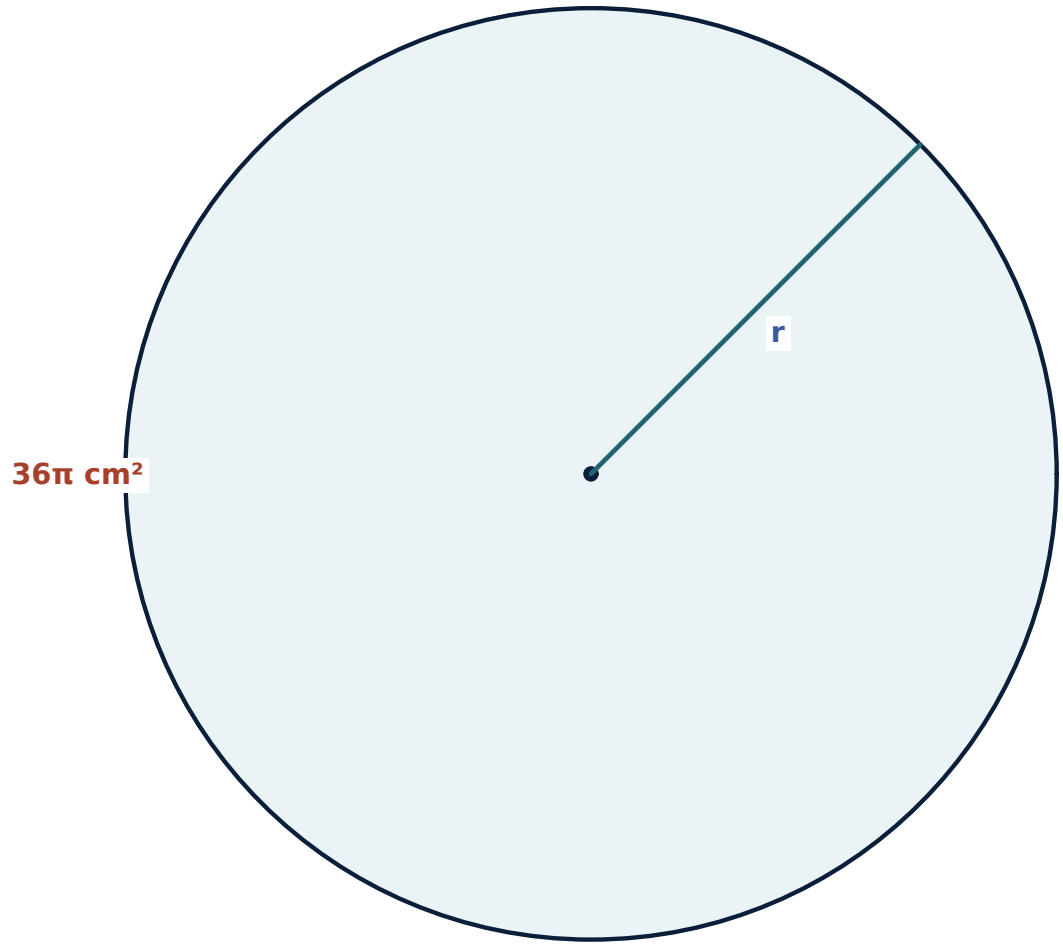
EXAMPLE A circle is drawn inside a square of side 10 cm, touching all four sides. Its radius is 5 cm, so it covers 25π cm² and the shaded corners are 100 − 25π = **21.5 cm²**. As a fraction of the square that is 1 − π÷4 = **21.5%** — and a 14 cm square with its own circle inside it gives 42.1 cm², which is 21.5% as well.

Concept 6 · Exact in π, then 3 significant figures — and backwards to the radius

When to keep π, when to round, and how to undo the formula

Two disciplines finish the unit. The first is **rounding**: keep π as a symbol through the working, use the π key and never 3.14, and round **once**, at the end, to 3 significant figures. The second is **running the formula backwards**. Given the area, divide by π first and then take the square root — in that order — and remember that a radius is a length, so it carries cm and not cm². If the question asks for the diameter, double your answer before you write it.

Running the formula backwards



Area given, radius wanted.

Substitute into $A = \pi r^2$ and the only thing left is r. Divide by π first, then take the square root — and remember that the answer is a length, so it is in cm and not cm^2.

Worked example 6 · Backwards, to a missing radius

1	A circular lid has an area of 36π cm². Write the formula with letters first.	**$A = \pi r^2$**
2	Substitute what you know.	**$36\pi = \pi r^2$**
3	Divide both sides by π.	**$r^2 = 36$**
4	Take the square root, and give the unit — a radius is a length, so it is not squared.	**r = 6 cm**
5	Now one that is not exact: a lid covers 50 cm². Divide by π first.	**$r^2 = 50 \div \pi = 15.9\ldots$**
6	Square-root it, and round once at the end.	**r = 3.99 cm**

Divide by π before you square-root, never after, and keep the full display on the calculator until the last line. Rounding r^2 to 15.9 first still gives 3.99 cm here, but it will not always.

What you press	What the screen shows	What you write	Verdict
π × 4²	50.2654824574	50.3 cm²	correct — the π key was used
3.14 × 4²	50.24	50.2 cm²	**wrong in the third figure** — 3.14 is not π
π × 7²	153.938040026	154 cm²	correct
49π − 9π	40π	126 cm²	correct — π was kept until the last line
154 − 28.3	125.7	126 cm²	risky — each piece was rounded first, so the total can be out

EXAMPLE A circular lid covers 50 cm². Divide by π first: $r^2 = 50 \div \pi = 15.915\ldots$ Then square-root it: r = **3.99 cm** to 3 significant figures. Keep the full display between the two steps, and notice the unit: the answer is a length, so it is cm, not cm².

The circle routine

1. Find the radius, whatever you were given — If the question gives a diameter, halve it before you do anything else and write the radius down. Half the mistakes in this topic are made in the first three seconds.

2. Split the shape, and name each piece — A compound shape is a list of pieces you already know. Write the list before you write any numbers, and mark whether each piece is being *added* or *taken away*.

3. Work out each piece, keeping π — Leave every area as a multiple of π while you add and subtract. 25π − 9π is 16π in one line, and no rounding error can creep into it.

✓ **Round once, at the end, and check the unit** — Only the final answer is rounded, to 3 significant figures, using the π key and not 3.14. Then check the unit is squared — and that the number is a sensible size for the picture.

The mistakes that cost the most marks

Trap 1 · Squaring the diameter instead of the radius

WRONG “The tank is 12 m across, so the base is π × 12^2 = 452 m^2.” The 12 was the diameter.

RIGHT Halve first: r = 6 m, and the area is π × 6^2 = **36π m^2**, which is **113 m^2**. Write “r = …” on its own line before you reach for the formula, every single time.

This error is always a factor of **four**, because doubling a length multiplies an area by 2^2. That makes it the easiest error in the whole strand to catch: if your circle covers more than the square it would fit inside, you have made it. A circle of diameter 12 m must be smaller than a 12 m by 12 m square, so 452 m^2 was never possible.

Trap 2 · Rounding π — or anything else — part-way through

WRONG “Area of a circle of radius 4 cm: 3.14 × 16 = 50.24, so 50.2 cm².” The third significant figure is wrong.

RIGHT Use the π key: π × 16 = 50.265…, so the area is **16π cm²**, which is **50.3 cm²** to 3 significant figures. Round **once**, at the end.

3.14 is only π to three significant figures itself, so an answer built on it can only be trusted to two. The safe habit is to keep π as a symbol right through the working — 49π − 9π = 40π has no rounding in it at all — and to press the π key only on the last line. The same rule covers every intermediate value, not just π.

Trap 3 · Counting a piece twice, or counting one that is not there

WRONG “A rectangle 20 cm by 10 cm has a semicircular end on each side, so the area is 200 + 25π + 25π = 357 cm².” Each end has been counted as a whole circle.

RIGHT Each end is **half** a circle of radius 5 cm, so the two together make **one** circle: 200 + 25π = **279 cm²**. Write the list of pieces down first, with a + or a − against each one, before any arithmetic starts.

The same slip in reverse is subtracting a hole that was never cut, or adding a piece that overlaps one you have already counted. The test is simple: every point of the shape must be inside **exactly one** piece you added, and every point you subtracted must have been inside something you added first. If two pieces overlap, the overlap has been counted twice and has to come off once.

Practice · Fluency · 24 questions

P1 A circle has a radius of 5 cm. **Calculate** its area, exactly and to 3 significant figures.

P2 A circle has a radius of 3 m. **Calculate** its area, exactly and to 3 significant figures.

P3 A circle has a diameter of 12 cm. **Calculate** its area, exactly and to 3 significant figures.

P4 A circle has a diameter of 10 m. **Calculate** its area, exactly and to 3 significant figures.

P11 A semicircle has a radius of 6 cm. **Calculate** its area, exactly and to 3 significant figures.

P12 A quarter circle has a radius of 8 cm. **Calculate** its area, exactly and to 3 significant figures.

P13 A semicircle has a diameter of 14 cm. **Calculate** its area, exactly and to 3 significant figures.

P14 A quarter circle has a radius of 6 m. **Calculate** its area, exactly and to 3 significant figures.

P21 A shape is a rectangle 20 cm by 10 cm with a semicircular end on each of the 10 cm sides. **Calculate** its area to 3 significant figures.

P22 A shape is a rectangle 12 m by 6 m with a semicircular end on one 6 m side. **Calculate** its area to 3 significant figures.

P23 A shape is a square of side 10 cm with a semicircle of diameter 10 cm on top. **Calculate** its area to 3 significant figures.

P24 A shape is a rectangle 6 cm by 4 cm with a quarter circle of radius 4 cm added at one end. **Calculate** its area to 3 significant figures.

P31 A ring has an outer radius of 7 cm and an inner radius of 3 cm. **Calculate** its area, exactly and to 3 significant figures.

P32 A ring has an outer radius of 10 cm and an inner radius of 6 cm. **Calculate** its area, exactly and to 3 significant figures.

P33 A ring has an outer radius of 5 cm and an inner radius of 4 cm. **Calculate** its area, exactly and to 3 significant figures.

P34 A circle of radius 8 cm has a circular hole of radius 2 cm cut from its centre. **Calculate** the area that is left, exactly and to 3 significant figures.

P41 A circle is drawn inside a square of side 10 cm, touching all four sides. **Calculate** the shaded area outside the circle to 3 significant figures.

P42 A circle is drawn inside a square of side 8 cm, touching all four sides. **Calculate** the shaded area outside the circle to 3 significant figures.

P43 A circle is drawn inside a square of side 20 cm, touching all four sides. **Calculate** the shaded area outside the circle to 3 significant figures.

P44 A circle of radius 4 cm is cut from a rectangle 12 cm by 8 cm. **Calculate** the area that is left to 3 significant figures.

P51 A circle has an area of 36π cm^2. **Determine** its radius.

P52 A circle has an area of 100π m^2. **Determine** its radius.

P53 **State** the area of a circle of radius 7 cm exactly, in terms of π.

P54 **Write** 16π cm^2 to 3 significant figures.

Practice · Applied · 24 questions

P5 A circular water tank has a base of diameter 12 m. **Calculate** the area of the base to 3 significant figures.

P6 A circle has a radius of 4 cm. **Calculate** its area exactly and to 3 significant figures, and **describe** why 3.14 is not good enough here.

P7 A round tablecloth has a radius of 0.9 m. **Calculate** its area, exactly and to 3 significant figures.

P8 A circle of radius 6 cm and a square of side 10 cm are drawn side by side. **Determine** which covers more, and by how much to 3 significant figures.

P15 A semicircular window has a diameter of 1.2 m. **Calculate** the area of the glass, exactly and to 3 significant figures.

P16 A quarter-circle flower bed of radius 5 m fills one corner of a yard. **Calculate** its area, exactly and to 3 significant figures.

P17 A semicircle and a quarter circle both have a radius of 4 cm. **Calculate** both areas to 3 significant figures and **describe** how they compare.

P18 A circular table of diameter 1.6 m is pushed against a wall so that exactly half of it is used. **Calculate** the area in use, exactly and to 3 significant figures.

P25 A window is a rectangle 1.5 m by 0.8 m with a semicircle of diameter 0.8 m on top. **Calculate** the area of the glass to 3 significant figures.

P26 A table top is a rectangle 20 cm by 10 cm with a semicircular end on each 10 cm side. **Calculate** its area to 3 significant figures and **describe** the shortcut the two ends allow.

P27 A concrete slab is a rectangle 4 m by 3 m with a semicircular end of radius 1.5 m on one 3 m side. **Calculate** its area to 3 significant figures.

P28 A shape is a rectangle 10 cm by 6 cm with a semicircle of radius 3 cm on each 6 cm side. **Calculate** its area to 3 significant figures.

P35 A steel washer has an outer diameter of 14 mm and an inner diameter of 6 mm. **Calculate** the area of metal, exactly and to 3 significant figures.

P36 A circular pond of radius 4 m is surrounded by a paved path 1 m wide. **Calculate** the area of the path, exactly and to 3 significant figures.

P37 A disc of radius 6 cm has a central hole of radius 1.5 cm. **Calculate** the area of the disc, exactly and to 3 significant figures.

P38 A square metal lid of side 20 cm has a circular hole of radius 5 cm cut in it. **Calculate** the area of metal to 3 significant figures.

P45 A circle is drawn inside a square so that it touches all four sides. **Determine** the fraction of the square that is *not* covered, exactly and as a percentage to 3 significant figures.

P46 A round table of diameter 1.2 m stands on a square mat of side 1.5 m. **Calculate** the area of mat left uncovered, to 3 significant figures.

P47 A quarter circle of radius 6 cm is cut from the corner of a square of side 6 cm. **Calculate** the area left to 3 significant figures, and **describe** what fraction of the square has gone.

P48 A circle of radius 3 cm is cut from the centre of a circle of radius 9 cm. **Determine** what fraction of the large circle is left, and give it as a percentage to 3 significant figures.

P55 A circle has an area of 50 cm². **Determine** its radius to 3 significant figures.

P56 A circular lawn is to cover 200 m². **Determine** the radius it needs, to 3 significant figures.

P57 A student finds the area of a circle of radius 4 cm using 3.14 and writes 50.2 cm². **Determine** the correct area to 3 significant figures and **describe** the error.

P58 A circle has an area of 81π cm². **Determine** its diameter.

Practice · Challenge · 12 questions

P9 One circle has a radius of 6 cm and another has a radius of 3 cm. **Determine** how many times bigger the first circle is, and **describe** the rule behind it.

P10 Two round tables have radii 0.6 m and 0.9 m. **Determine** how much more surface the larger one has, exactly and to 3 significant figures.

P19 A semicircle of radius 6 cm has the same area as a quarter circle. **Determine** the radius of the quarter circle, exactly and to 3 significant figures.

P20 A quarter circle of radius 12 cm is cut from a square of side 12 cm. **Determine** the area that is left, to 3 significant figures.

P29 A shape is a square of side 12 cm with a semicircle of diameter 12 cm on top. **Determine** its area to 3 significant figures and **describe** how you would check the answer is sensible.

P30 A rectangle 20 cm by 10 cm with a semicircular end on each 10 cm side has the same area as a circle. **Determine** that circle’s radius to 3 significant figures.

P39 A ring has an area of 40π cm² and an outer radius of 7 cm. **Determine** its inner radius.

P40 Washer A has radii 8 cm and 6 cm; washer B has radii 7 cm and 4 cm. **Determine** which has more metal, and by how much, exactly and to 3 significant figures.

P49 A circle is drawn inside a square of side 14 cm, touching all four sides. **Determine** the shaded area to 3 significant figures, and **describe** why it is about a fifth of the square.

P50 Three circles of radius 2 cm are cut from a strip of card 12 cm by 4 cm. **Determine** the area of card left to 3 significant figures.

P59 A circle has an area of 154 cm², correct to 3 significant figures. **Determine** its diameter to 3 significant figures.

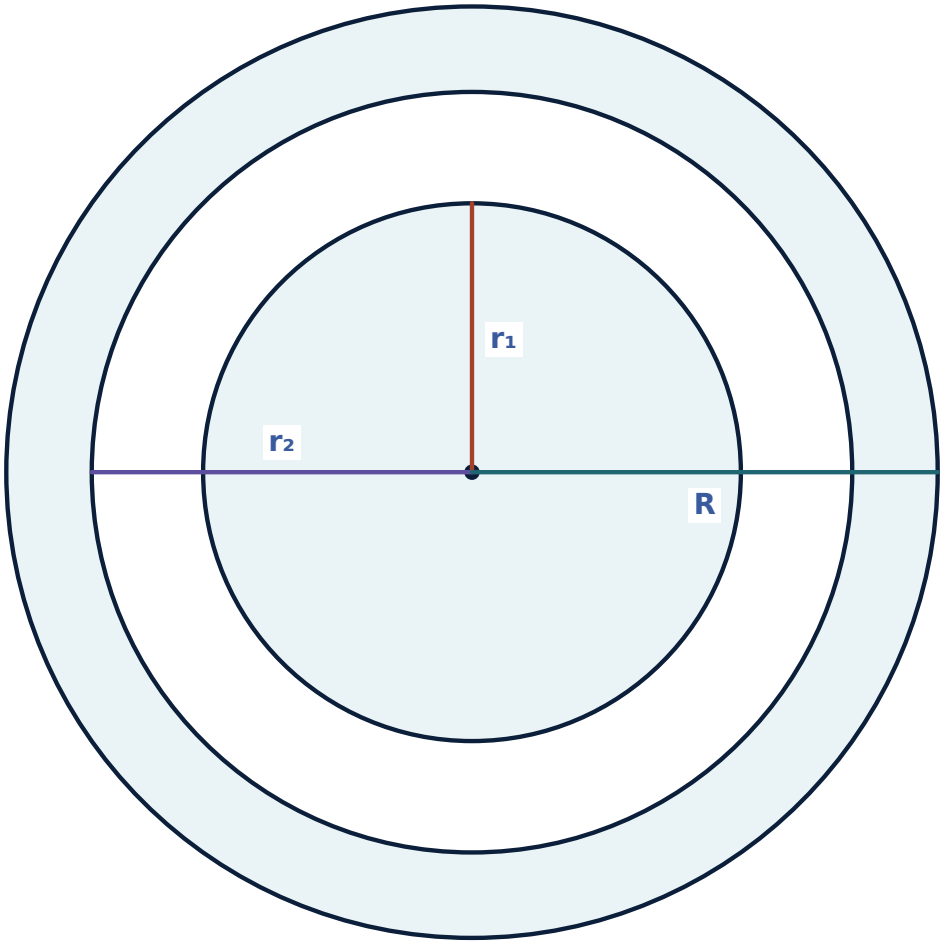
P60 A circular tank base has an area of 12π m². It is replaced by one of exactly twice the area. **Determine** the new radius, exactly and to 3 significant figures.

Criterion B · Investigating patterns

Rings of equal area

A target is painted as a disc with rings drawn on it, and the painter wants something unusual: **every ring is to have exactly the same area as every other one**, so that each takes the same amount of paint. That is not the same as making the rings the same width — and the difference is the whole investigation. The disc has a radius of 6 cm. The question is **where the circles have to be drawn**, and whether there is a rule that puts them there for any number of rings at all. You need a compass, a ruler and a calculator, and you should give every radius exactly first and then to 3 significant figures.

A disc cut into three rings of equal area



The three rings have exactly the same area as each other, which is why they do not have the same width. Where do the two inside circles have to be drawn?

Part 1 · structured

Rings n	Radius of each circle, from the middle outwards	The same radii to 3 s.f.	Area of each ring
2	3√2 cm, 6 cm	4.24 cm, 6.00 cm	18π cm²
3			
4			

1. **Predict**, before you calculate anything, whether the middle circle of a two-ring target sits at half the radius, more than half, or less than half. Write the prediction down; you will be asked to compare it with what you find.

2. **Complete** the table for n = 2, 3 and 4 rings on a disc of radius 6 cm. For each one, work out where each circle has to be drawn and check that the rings really do come out equal by working out the area of each ring — remember that a ring is the circle out to it minus the circle inside it. Give each radius exactly and to 3 significant figures.

3. **Identify** the pattern. Look at the disc out to the k-th circle rather than at the rings themselves, and **describe** what fraction of the whole disc that inner disc has to be. Then **describe** what that says about r_1 compared with R.

4. **Determine** a general rule for the radius of the k-th circle when a disc of radius R is divided into n rings of equal area. Write it as a formula in R, n and k, not in words alone.

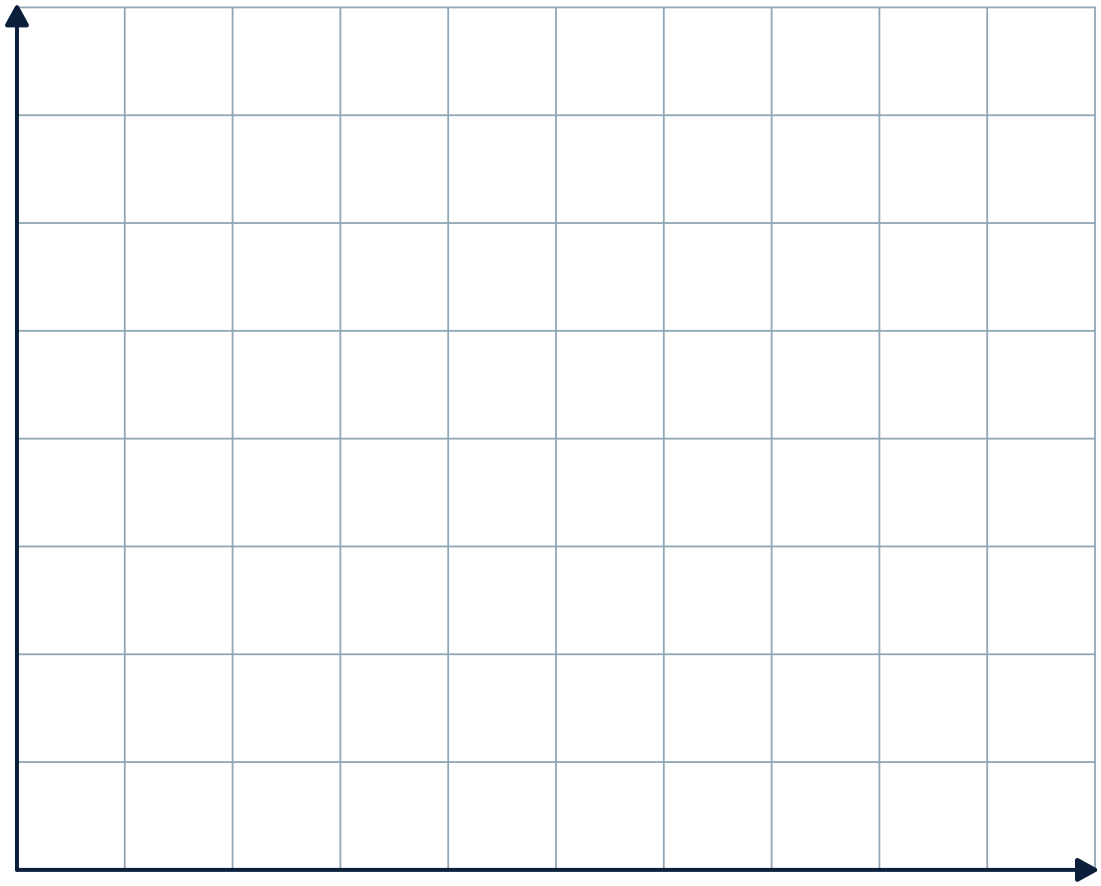
5. **Test and verify** your rule on two cases that are not in the table. First, predict all six radii for a target with n = 6 rings on the same 6 cm disc, then check by working out the area of at least two of the rings. Second, use the blank grid to draw a target of your own with a different radius, and check that one of its rings really has the area your rule says it should.

6. **Justify** your rule. **Describe** why the disc out to the k-th circle must be exactly k parts out of n of the whole disc, and show how that gives your formula in two lines of algebra. Then **describe** why the rings get narrower as they go outwards, even though their areas are all the same.

Your own target · the blank grid for step 5

Sketch a target of your own below, with a radius you choose. Mark the centre first so that every circle really is concentric, work out where your rule says each circle has to go, and then check that one of the rings really has the area your rule promised.

Your own target



Squared paper for your own target, drawn to your own rule.

Use this grid to sketch a target of your own, mark the radii your rule gives, and check that the rings really do come out equal before you write the rule down.

Part 2 · open

Now you choose what to investigate. Take one of these openings or propose your own. You decide which cases to try, how many you need, how to record what you find and how you will test it.

1. Equal widths instead. Draw a target whose rings all have the same **width** rather than the same area. **Investigate** what fraction of the disc each ring then takes, **determine** a rule for the k-th ring, and **describe**

which ring is the biggest and why.

2. A bullseye that counts. Suppose the middle circle of a target has to be worth exactly a tenth of the whole disc, and the rest of the disc is then split into rings of equal area. **Investigate** where the circles go, **determine** a rule, and **describe** how it is related to the one you already have.

3. Scaling the target. Take one of your targets and double R. **Investigate** what happens to every radius, to every ring area and to the total area. **Predict** what tripling R would do, **determine** a rule, and **describe** the connection with $A = \pi r^2$.

Whichever you choose, take it through the same ladder: **predict**, **identify** the pattern, **determine** a general rule, **test and verify** it on cases you have not already worked out, and **justify** why it must be true.

Marked out of 8 against Criterion B. Completing the table earns the lower bands; the general rule, the test on the six-ring target and on a target of your own, and the justification are what reach levels 5–8. Part 2 must show your own choice of method.

Remember, you should communicate in an organized and coherent manner.

Criterion D · Applying maths in real-life contexts

How much of the field does the pivot actually water?

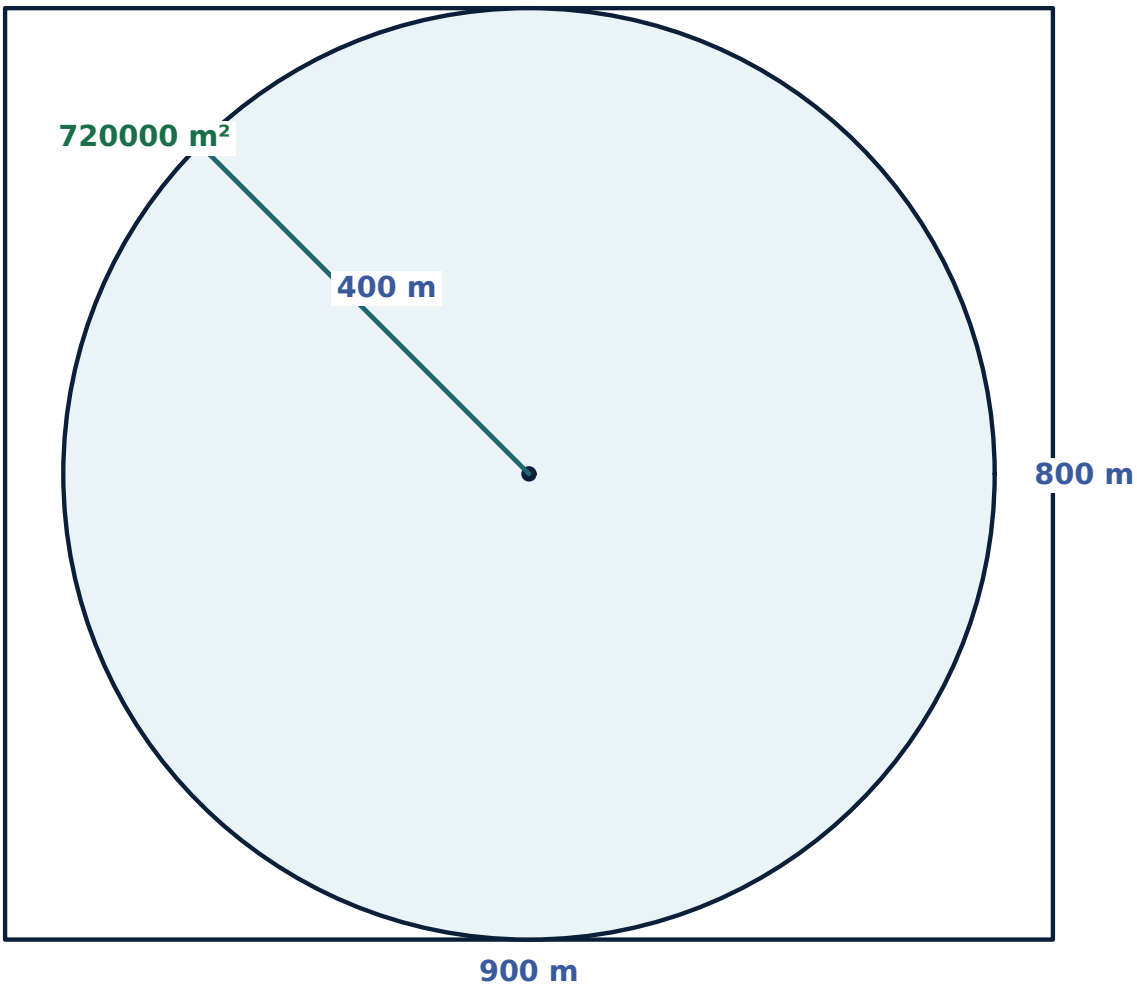
A farm in the Tuli Block has one rectangular field and one centre-pivot irrigation machine. The machine stands in the middle and its arm sweeps right round, so **the watered land is a circle inside a rectangle** — and the corners of the field are never reached. The plan sheet below **says a great deal more than you need**. Two questions have to be answered before the farmer decides whether to plant the corners with something else: **what fraction of the field does the pivot actually water**, and **how much land is left dry**.

The plan sheet (illustrative figures, written for this task)

- **The field:** a rectangle, 900 m long and 800 m wide
- **The pivot:** one centre-pivot machine, standing at the middle of the field, with an arm 400 m long
- **What it waters:** a circle, as the arm turns right round
- **A full turn:** 22 hours
- **Water applied:** 12 mm on each pass
- **Towers:** seven, on rubber tyres
- **The borehole:** tested at 45 000 litres an hour
- **The crop:** maize, planted in November
- **Fencing:** 3 400 m of the boundary is already fenced
- **Farm track:** a 4 m track runs along one of the long sides

These figures are illustrative. The farm, the field and the plan sheet are an invented illustrative example written for the task. It is not taken from any business, price list or publication, and no organisation, person or source is named or implied. What is authentic is the shape of the problem: a centre-pivot machine really does water a circle, and the corners of a rectangular field really are left dry.

The field, and the circle the pivot can reach



The pivot turns about the middle of the field.

The machine turns about the centre and waters a circle. The field is a rectangle, so some of it is never reached: two strips at the ends, and four corner pieces. Which of those is the bigger loss?

Identify the entries on the plan that actually decide the two answers, and **describe**, item by item, why the others cannot affect either one — deal with the 22-hour turn, the 12 mm of water, the seven towers, the borehole, the crop, the fencing and the track by name. *(Strand i — identifying the relevant elements)*

Select the formulas you need and **describe** why each is the right one. Say how you will find the dry land: whether by taking the circle away from the whole field in one step, or by splitting the dry land into the two end strips and the four corners and adding them — and say which you will use as the check on the other. *(Strand ii — selecting appropriate strategies)*

Apply them. **Calculate** the area of the field, the area the pivot waters, the fraction of the field that is watered, the area left dry, the area of the two end strips, the area of the four corners together and the area of one corner. Give every area in square metres and again in hectares, where 1 hectare is 10 000 m². *(Strand iii — applying the strategy)*

Describe the accuracy of your answers. The watered area is exactly 160 000π m² and nothing else here needs rounding at all, so **describe** where the rounding actually happens, why you kept π until the last line, and what would have gone wrong if you had used 3.14 from the start. Then **describe** how your two routes to the dry land checked each other. *(Strand iv — accuracy of the solution)*

Describe whether your answers make sense for a real farm. **Describe** at least two things the calculation ignores that a farmer would not, say whether the corners are worth planting with something else, and say what you would advise — with the number that supports it. *(Strand v — does it make sense in context)*

Submit: your list of which entries matter and which do not, the area of the field, the watered area, the fraction watered, the dry area, the two end strips, the four corners together and one corner — each in square metres and in hectares — and written answers for strands i, iv and v. **Marked out of 8** against Criterion D at Foundations level, where the command term is **describe**. Keep π in the working and give any answer that is not exact to 3 significant figures.

Answers

How the P-numbers run. The 60 practice items are numbered P1 to P60 in the order they appear in the lesson — ten per concept, and inside each concept four Fluency, then four Applied, then two Challenge. The same P-number identifies the item in the lesson, in this key and in the teacher pack, so the numbers inside each tier below step in blocks of ten rather than running consecutively: Fluency is P1–P4, P11–P14, P21–P24 and so on; Applied is P5–P8, P15–P18, …; Challenge is P9–P10, P19–P20, … Areas carry a squared unit and lengths do not, because a radius is a length. Where an answer is exact in terms of π it is given that way first, and then to 3 significant figures.

Fluency · F-tier (24)

P1	**25π cm², or 78.5 cm²**
P2	**9π m², or 28.3 m²**
P3	**36π cm², or 113 cm²**
P4	**25π m², or 78.5 m²**
P11	**18π cm², or 56.5 cm²**
P12	**16π cm², or 50.3 cm²**
P13	**24.5π cm², or 77.0 cm²**
P14	**9π m², or 28.3 m²**
P21	**279 cm²**
P22	**86.1 m²**
P23	**139 cm²**
P24	**36.6 cm²**
P31	**40π cm², or 126 cm²**
P32	**64π cm², or 201 cm²**
P33	**9π cm², or 28.3 cm²**
P34	**60π cm², or 188 cm²**

P41	**21.5 cm²**
P42	**13.7 cm²**
P43	**85.8 cm²**
P44	**45.7 cm²**
P51	**6 cm**
P52	**10 m**
P53	**49π cm²**
P54	**50.3 cm²**

Applied — fully worked (24)

P5 113 m²

The formula wants the radius, so halve the diameter first: r = 6 m. Then A = πr² = π × 36 = 36π m², which is 113 m² to 3 significant figures. Squaring the 12 would give 452 m², four times too much.

P6 16π cm², or 50.3 cm²

A = π × 4² = 16π cm². With the π key that is 50.265…, which rounds to 50.3 cm². Using 3.14 gives 50.24, which rounds to 50.2 cm² and is wrong in the third figure. 3.14 is itself only π to three figures, so it cannot support a three-figure answer.

P7 0.81π m², or 2.54 m²

A = π × 0.9². Square the radius first: 0.9 × 0.9 = 0.81, so the area is 0.81π m², which is 2.54 m² to 3 significant figures. Doubling 0.9 to get 1.8 is the slip to avoid — squaring is not doubling.

P8 the circle, by 13.1 cm²

The circle is π × 6² = 36π cm², which is 113 cm². The square is 10 × 10 = 100 cm². The circle is larger, and the difference is 36π − 100 = 13.1 cm² to 3 significant figures. Working the difference out exactly first, and rounding once, is safer than subtracting two rounded values.

P15 0.18π m², or 0.565 m²

Halve the diameter: r = 0.6 m. The whole circle would be π × 0.36 = 0.36π m², so the semicircle is half of that, 0.18π m², which is 0.565 m² to 3 significant figures. Work out the whole circle first, then halve it — halving

the radius instead would be quite wrong.

P16 6.25π m², or 19.6 m²

The whole circle would be π × 5² = 25π m². A quarter of that is 25π ÷ 4 = 6.25π m², which is 19.6 m² to 3 significant figures. The two straight edges of the bed are radii, and they play no part in an area calculation.

P17 25.1 cm² and 12.6 cm²; the semicircle is exactly twice the quarter circle

The whole circle is 16π cm². The semicircle is half of it, 8π cm² = 25.1 cm², and the quarter circle is a quarter of it, 4π cm² = 12.6 cm². A half is twice a quarter, so the first is twice the second — and comparing them in terms of π makes that obvious, while comparing the rounded decimals hides it.

P18 0.32π m², or 1.01 m²

The radius is 0.8 m, so the whole table is π × 0.64 = 0.64π m². Half of it is 0.32π m², which is 1.01 m² to 3 significant figures. Note that 0.8² = 0.64, not 1.6 — squaring a decimal less than one makes it smaller.

P25 1.45 m²

The rectangle is 1.5 × 0.8 = 1.2 m². The semicircle has radius 0.4 m, so it is ½ × π × 0.16 = 0.08π m². Adding: 1.2 + 0.08π = 1.45 m² to 3 significant figures. The pieces meet along the top of the rectangle and do not overlap.

P26 279 cm²; the two ends make one whole circle

The rectangle is 200 cm². Both ends are semicircles of radius 5 cm, so together they are one circle of area 25π cm². The total is 200 + 25π = 279 cm² to 3 significant figures. Counting each end as a whole circle would give 357 cm², which is the commonest error in the topic.

P27 15.5 m²

The rectangle is 4 × 3 = 12 m². The semicircle has radius 1.5 m, so it is ½ × π × 2.25 = 1.125π m². The total is 12 + 1.125π = 15.5 m² to 3 significant figures. The radius 1.5 m is half of the 3 m side, which is what makes the end fit.

P28 88.3 cm²

The rectangle is 60 cm². The two semicircles of radius 3 cm together make one circle, π × 9 = 9π cm². The total is 60 + 9π = 88.3 cm² to 3 significant figures. Check the size: the answer must be bigger than the rectangle and smaller than a 16 cm by 6 cm box, and 88.3 sits between 60 and 96.

P35 40π mm², or 126 mm²

Halve both diameters first: the radii are 7 mm and 3 mm. The whole disc is 49π mm² and the hole is 9π mm², so the metal is 49π − 9π = 40π mm², which is 126 mm² to 3 significant figures. Two diameters means two halvings, and forgetting one of them is the usual slip.

P36 9π m², or 28.3 m²

The path is a ring. Its outer radius is 4 + 1 = 5 m, and its inner radius is the pond's 4 m. The area is 25π − 16π = 9π m², which is 28.3 m² to 3 significant figures. The path's width is added to the **radius**, not to the diameter, and never multiplied by anything.

P37 33.75π cm², or 106 cm²

The whole disc is π × 36 = 36π cm² and the hole is π × 2.25 = 2.25π cm². Subtracting in terms of π: 36π − 2.25π = 33.75π cm², which is 106 cm² to 3 significant figures. Note $1.5^2 = 2.25$, not 3.

P38 321 cm²

The lid is 20 × 20 = 400 cm² and the hole is π × 25 = 25π cm². The metal is 400 − 25π = 321 cm² to 3 significant figures. The hole method does not care that one shape is straight-edged and the other is round — whole minus hole is the whole of it.

P45 1 − π÷4, which is 21.5%

Call the side s. The circle's radius is s ÷ 2, so its area is $\pi s^2 \div 4$. The fraction covered is $\pi s^2 \div 4 \div s^2 = \pi \div 4$, and the s^2 cancels — so the answer does not depend on the size at all. What is left is 1 − π ÷ 4 = 0.2146…, which is 21.5% to 3 significant figures.

P46 1.12 m²

The mat is 1.5 × 1.5 = 2.25 m². The table has radius 0.6 m, so it covers π × 0.36 = 0.36π m². Uncovered: 2.25 − 0.36π = 1.12 m² to 3 significant figures. The table does fit, because its 1.2 m width is less than the mat's 1.5 m side.

P47 7.73 cm²; π÷4 of the square, about 78.5%, has gone

The square is 36 cm² and the quarter circle is ¼ × 36π = 9π cm². What is left is 36 − 9π = 7.73 cm² to 3 significant figures. The fraction removed is 9π ÷ 36 = π ÷ 4, the same fraction as a circle inscribed in a square, which is 78.5%.

P48 8 ÷ 9 of it, which is 88.9%

The circles are 81π cm² and 9π cm², so what is left is 72π cm². The fraction is 72π ÷ 81π = 8 ÷ 9, because the π cancels, and that is 88.9% to 3 significant figures. Cutting a hole of a third of the radius removes only a

ninth of the area.

P55 3.99 cm

Substitute into $A = \pi r^2$: $50 = \pi r^2$. Divide by π first: $r^2 = 50 \div \pi = 15.915\ldots$ Then take the square root: r = 3.99 cm to 3 significant figures. Divide before you root, never after, and keep the full display until the last line.

P56 7.98 m

$200 = \pi r^2$, so $r^2 = 200 \div \pi = 63.66\ldots$ and r = 7.98 m to 3 significant figures. A sense check: a 16 m by 16 m square is 256 m^2, and the lawn is a bit smaller than that square, which fits.

P57 50.3 cm^2

With the π key, $\pi \times 16 = 50.265\ldots$, which is 50.3 cm^2 to 3 significant figures. Using 3.14 gives 50.24 and so 50.2 cm^2. The error is that 3.14 is itself only π to three significant figures, so any answer built on it can be trusted to two at best. Use the π key, or keep π as a symbol.

P58 18 cm

Divide by π: $r^2 = 81$, so r = 9 cm. The question asks for the diameter, which is twice the radius: d = 18 cm. Answering 9 cm here is the commonest lost mark in the whole topic — read the last line of the question again before you write.

Challenge — fully worked (12)

P9 4 times bigger

The areas are 36π cm^2 and 9π cm^2, and $36\pi \div 9\pi = 4$. Doubling the radius does not double the area, it multiplies it by $2^2 = 4$, because the radius is squared in the formula. Three times the radius would give nine times the area.

P10 0.45π m^2, or 1.41 m^2

The areas are 0.36π m^2 and 0.81π m^2. Subtract while π is still there: $0.81\pi - 0.36\pi = 0.45\pi$ m^2, which is 1.41 m^2 to 3 significant figures. Rounding each area first would give $2.54 - 1.13 = 1.41$ as well here, but it is not always so forgiving.

P19 6√2 cm, or 8.49 cm

The semicircle is 18π cm^2. For the quarter circle, $\frac{1}{4}\pi r^2 = 18\pi$, so $r^2 = 72$ and $r = \sqrt{72} = 6\sqrt{2}$ cm, which is 8.49 cm to 3 significant figures. The π cancels on both sides, which is why keeping it as a symbol makes this

question short.

P20 30.9 cm²

The square is 12 × 12 = 144 cm². The quarter circle is ¼ × π × 12² = 36π cm². What is left is 144 − 36π cm², which is 30.9 cm² to 3 significant figures. The quarter circle fits exactly into the corner, so nothing overlaps.

P29 201 cm²

The square is 144 cm² and the semicircle has radius 6 cm, so it is ½ × 36π = 18π cm². The total is 144 + 18π = 201 cm² to 3 significant figures. The check: the semicircle sits inside a 12 cm by 6 cm rectangle of area 72 cm², so the answer has to lie between 144 and 216 cm² — and it does.

P30 9.42 cm

The compound shape is 200 + 25π cm². For the circle, πr² = 200 + 25π, so r² = 200 ÷ π + 25 = 88.66… and r = 9.42 cm to 3 significant figures. Keep the whole display on the calculator between the two steps: rounding r² to 88.7 first would still give 9.42 here, but the habit is what protects you elsewhere.

P39 3 cm

The ring is the whole disc minus the hole: 49π − πr² = 40π. Dividing everything by π gives 49 − r² = 40, so r² = 9 and r = 3 cm. Dividing by π at the start turns a messy equation into a one-line one, and the answer is a length, so it is not squared.

P40 washer B, by 5π cm², or 15.7 cm²

Washer A is 64π − 36π = 28π cm² and washer B is 49π − 16π = 33π cm². So B has more, by 33π − 28π = 5π cm², which is 15.7 cm² to 3 significant figures. A has the bigger outside, which is why the answer is worth checking rather than guessing.

P49 42.1 cm², which is 21.5% of the square

The square is 196 cm² and the circle has radius 7 cm, so it is 49π cm². Shaded: 196 − 49π = 42.1 cm² to 3 significant figures. As a fraction that is 1 − π ÷ 4 = 21.5%, the same as for the 10 cm square, because the fraction never depends on the size — a fifth is a good mental estimate for any of them.

P50 10.3 cm²

The strip is 12 × 4 = 48 cm². Each circle is π × 4 = 4π cm², so three of them are 12π cm². What is left is 48 − 12π = 10.3 cm² to 3 significant figures. The three circles fit exactly, because each is 4 cm across and 3 × 4 = 12.

P59 14.0 cm

$r^2 = 154 \div \pi = 49.019\ldots$, so $r = 7.0014\ldots$ cm and $d = 14.0$ cm to 3 significant figures. The trailing zero has to be written: “14” claims two significant figures, “14.0” claims three. Note also that the area was itself already rounded, so the diameter cannot be trusted beyond three figures either.

P60 2√6 m, or 4.90 m

The new area is 24π m². Dividing by π gives $r^2 = 24$, so $r = \sqrt{24} = 2\sqrt{6}$ m, which is 4.90 m to 3 significant figures. Doubling the area does **not** double the radius: it multiplies it by √2, from 3.46 m to 4.90 m.