Geometry & Trigonometry · Unit G15 · Standard

The Gradient Game

Standard · MYP 4–5 · 60 practice questions · Criteria A, B, C, D

The fifteenth unit of the Geometry & Trigonometry strand, and the fifth of the Standard band. G09 gave every point an address and stopped at counting. This unit joins two points up and asks how steep the join is — the one number that turns a picture of a line into an equation, and the one that decides whether two lines ever meet.

A road sign that says 1 in 8, a wheelchair ramp that has to rise no more than 1 in 12, a roof pitched so the rain runs off, a graph of fuel against distance — every one of them is the same idea wearing different clothes. **Steepness is a number**, and you get it by asking how far the thing goes up for every step it goes along. This unit is that number: how to read it off a picture, how to draw a picture from it, and why two lines with the same one can travel forever without ever meeting.

By the end of this unit you can

• **Calculate** the gradient of a line from two points on it, and **state** the gradient of a horizontal line and why a vertical line has none

• **Determine** m and c from a line drawn on a grid, using a rise and run triangle taken between two crossings

• **Draw** a line from its equation by finding where it crosses each axis, and **verify** it with a third point

• **Determine** the equation of the line through two given points, and **write down** the equation of a horizontal or a vertical line

• **Determine** the equation of the line parallel to a given line through a given point

• **Identify** which of several equations describe parallel lines, and **explain** why sharing a y-intercept has nothing to do with it

The one idea behind all of it

The **gradient** of a straight line is how far it rises for every one unit it runs, and it is the same number wherever on the line you measure it — that is what makes the line straight. Take any two points on it, count the **run** across and the **rise** up, and divide: **m = rise ÷ run**. A line that falls has a negative rise and so a negative gradient. A flat line rises nothing, so its gradient is 0. An upright line runs nothing at all, and since you may not divide by zero its gradient is **undefined**. Put that number together with the height at which the line crosses the y-axis and you have the line's whole address: **y = mx + c**.

m = rise ÷ run, and c is where the line crosses the y-axis. Two habits do most of the work: count the run and the rise off the axes and never off the page, because a grid square need not be drawn square; and check the sign against the picture, because a line that falls must have a negative gradient.

A gradient carries no unit; everything else does. A gradient is a length divided by a length, so the units cancel. A distance is given in units or metres, an area in square units, and a rate read off a real graph carries the unit of the vertical axis over the unit of the horizontal one. Anything that is not exact is given to 3 significant figures. Round, never truncate.

Where this unit sits

This is the picture half of the straight line. Unit G09 *Find the Point* gave every point an address and stopped at counting; G15 joins those points up and asks how steep the join is. The distance between two points that do *not* share a row or a column, and the midpoint of the segment between them, are **G16 Distance, Midpoint, Gradient**, which comes next and uses this unit's gradient as one of its three formulae. The **perpendicular** rule — what happens to a gradient when a line is turned through a right angle — is **G24**, in the Extended band, and is deliberately not here. Writing a relationship in letters and substituting a value into it are **Algebra u05 Say It In Algebra** and **Algebra u07 Formula Power**; y = mx + c is one more formula of that kind, and neither of those units is retaught on this page either.

A gradient is the one number in this strand that carries no unit. It is a length divided by a length, so the units cancel: a gradient of 2 is 2 whether the grid is ruled in centimetres or in kilometres. Every other number in the unit still carries its unit — a distance in units or metres, a

fuel figure in litres. A *rate* read off a real graph does carry one, and it is the unit of the vertical axis over the unit of the horizontal one: litres per 100 km, pula per hour, metres per second. That is the rate-as-a-ratio idea of **N11 Sharing Fairly**, read off a graph instead of out of a sentence.

House rules. An equation is written **y = mx + c** with the true minus sign: y = 2x − 3, never y = 2x + −3. A gradient that is not a whole number is written as a decimal where the decimal is exact — 0.5, −1.5 — and to 3 significant figures where it is not, rounded and never truncated. A coordinate pair is written in brackets with a comma: (0, 5).

Note · the bridge to Algebra u18

You have met y = mx + c before, and this unit does not teach it again. The official topic grid files two different rows under two different headings. Under *Algebra* sits the machinery: rearranging an equation into the form y = mx + c, solving two equations together to find where their lines cross, and reading a pair of simultaneous equations as a pair of lines. All of that is built and live in **Algebra u18 The Straight Line**, and nothing on this page repeats it. Under *Spatial reasoning* — which is what this strand covers — sits the **picture**: the line as a drawing on the plane, the gradient read off the geometry with a rise and run triangle, the two crossings that let you draw it, and the fact that parallel lines have equal gradients. That, and only that, is G15. If a question here asks you to rearrange something awkward or to find where two lines cross, it is borrowing from u18 on purpose and will say so.

Check you are ready. Five questions from the u18 bank, with their answers. If all five come out, this unit will make sense from the first line. If two or more do not, go back to u18 first — nothing here is going to teach them to you again.

Check you are ready	Answer	Why
State the gradient of y = 5x − 3.	5	In y = mx + c the gradient is the number multiplying x, and it keeps the sign in front of it.
State the y-intercept of y = 5x − 3.	−3	The intercept is c, the number on its own — and the minus sign belongs to it.

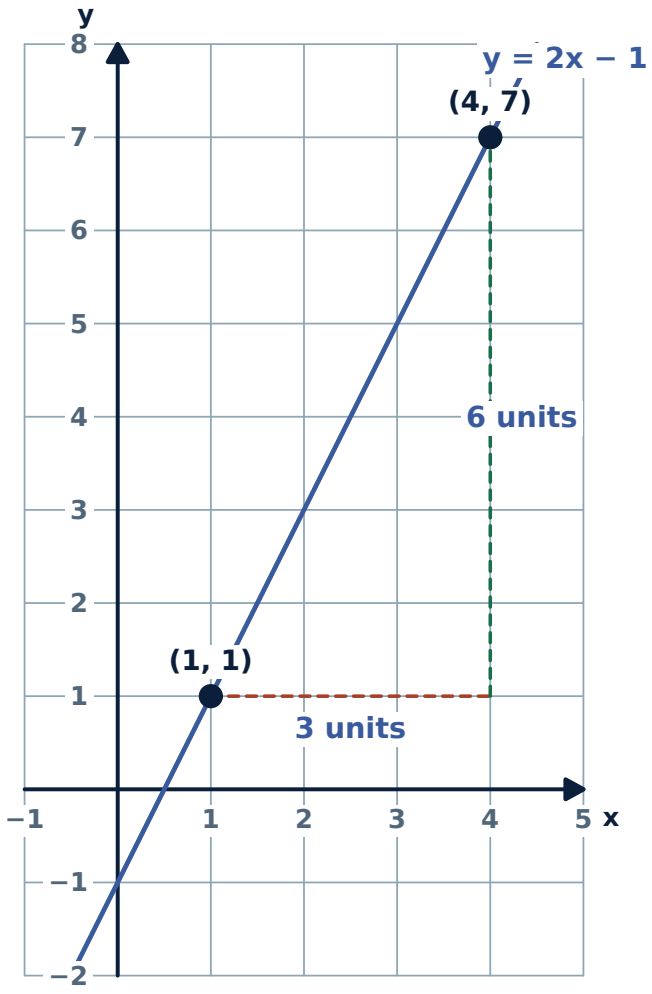
Check you are ready	Answer	Why
Calculate the gradient of the line through (1, 4) and (5, 16).	3	The run is 4 and the rise is 12, so the gradient is 12 divided by 4.
Write down the equation of the line parallel to y = 3x + 1 that passes through (0, 7).	y = 3x + 7	Parallel means the same gradient, so m stays at 3; the point given is on the y-axis, so it hands you c directly.
A line has gradient 4 and passes through (0, −5). **Write down** its equation.	y = 4x − 5	m is given and the point is the y-intercept itself, so c is −5.

Concept 1 · Gradient: rise over run

How steep, and which way — including the flat line and the upright one that has no gradient at all

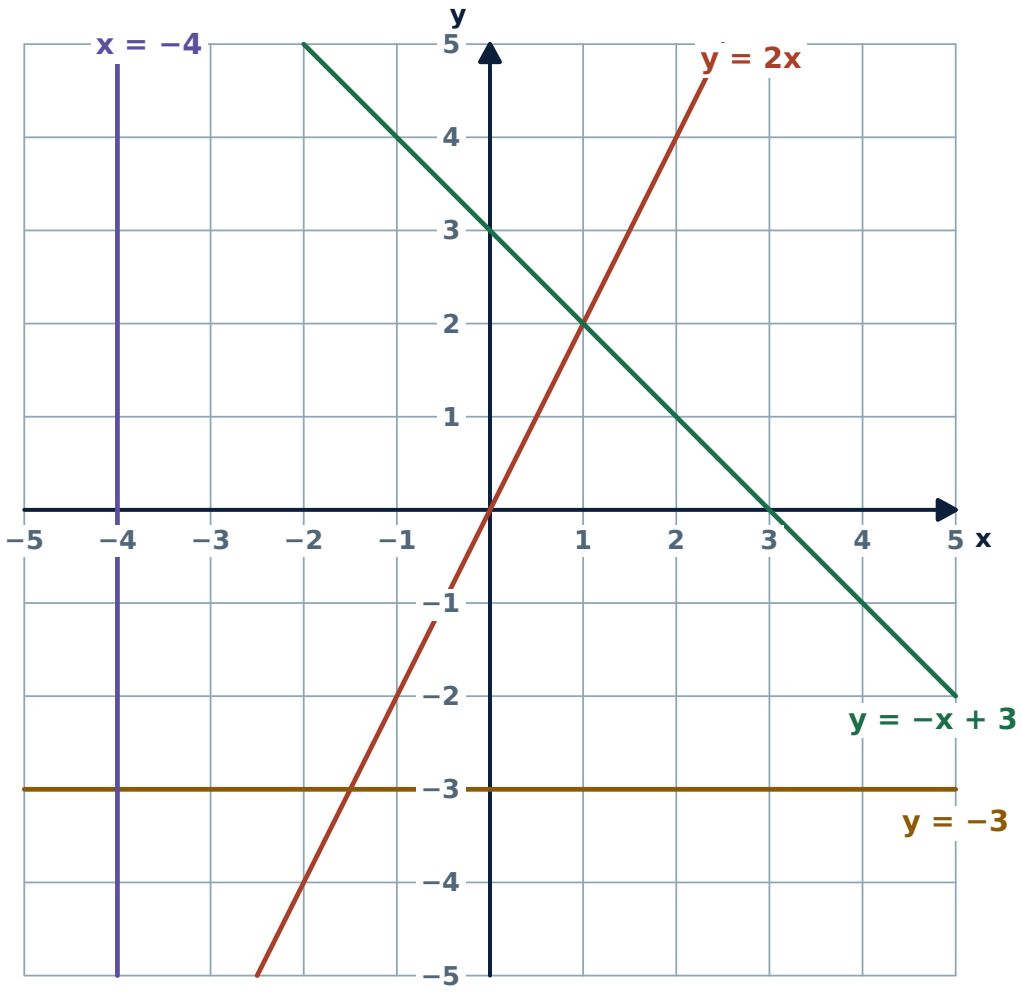
The **gradient** of a line is how far it rises for every one unit it runs, and it is the same number wherever on the line you measure it. Pick two points, count the **run** along and the **rise** up — keeping the sign of each — and divide: **m = rise ÷ run**. Four cases cover everything. A line that climbs has a positive gradient; one that falls has a negative one; a flat line rises nothing, so m is 0; and an upright line runs nothing, so its gradient is **undefined** rather than zero.

Rise over run, drawn as a triangle



The run is measured first and the rise second, exactly as a coordinate pair is read. Here the line goes 3 units along and 6 units up, so the gradient is 6 divided by 3, which is 2 — and it would be 2 whichever pair of points on the line you chose.

Four lines, four kinds of gradient



A rising line has a positive gradient and a falling line a negative one. A horizontal line has a gradient of zero, because it rises nothing at all. A vertical line has no gradient: its run is zero, and dividing by zero is not something you are allowed to do.

Worked example 1 · Rise over run, and the four kinds

1	Take the two marked points on the line, (1, 1) and (4, 7). Count the run first.	**3 along**
2	Now count the rise, from the end of the run up to the second point.	**6 up**
3	**Calculate** the gradient: rise divided by run.	**m = 2**
4	Choose a different pair of points on the same line and do it again.	**still 2**
5	**Explain** what happens to the rise when the line falls instead. Take (0, 6) to (4, 0).	**rise −6, so m = −1.5**
6	**State** the gradient of a flat line, and of an upright one.	**0, and undefined**

The gradient is the same number wherever you measure it, which is exactly what being straight means. A falling line has a negative rise and therefore a negative gradient; a flat line rises nothing so m is 0; an upright line runs nothing, and since you cannot divide by zero it has no gradient at all.

Two points on the line	Run	Rise	Gradient	What it does
(0, 0) to (1, 2)	1	2	2	climbs steeply
(0, 0) to (4, 2)	4	2	0.5	climbs gently
(0, 6) to (4, 0)	4	−6	−1.5	falls
(−3, 2) to (5, 2)	8	0	0	stays flat
(4, −1) to (4, 6)	0	7	undefined	goes straight up

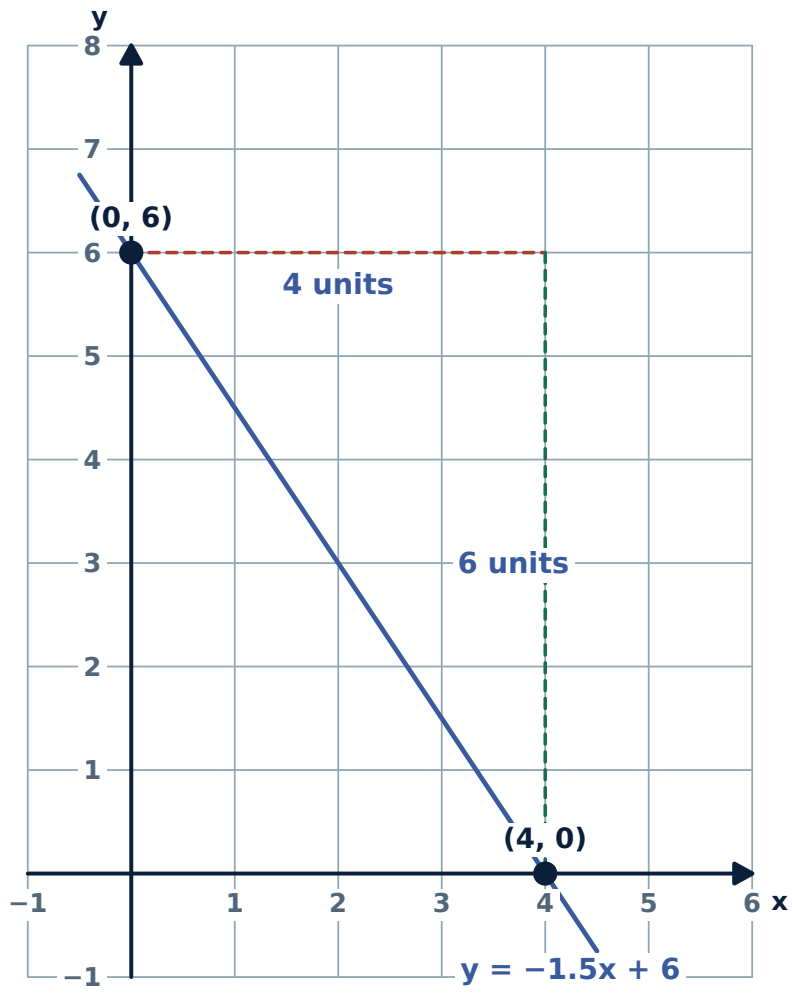
EXAMPLE From (1, 1) to (4, 7) the run is 3 and the rise is 6, so the gradient is **2**. From (0, 6) to (4, 0) the run is 4 but the line drops, so the rise is −6 and the gradient is **−1.5**. Same arithmetic, one minus sign.

Concept 2 · Reading m and c off a drawn line

The y-intercept read off the axis, the gradient read off a triangle drawn between two crossings

A line drawn on a grid hands you both of its numbers if you know where to look. **c** is not calculated at all: it is the y-value at which the line crosses the y-axis, and you read it straight off the axis. **m** comes from a triangle you draw yourself between two crossings the line passes exactly through — count the run, count the rise, divide, and then check the sign against the line you can see. Put the two numbers into y = mx + c and the line has been written down.

Reading m and c straight off the picture



The line crosses the y-axis at 6, so c is 6. Going 4 units along it drops 6 units, so the gradient is negative 6 over 4, which is −1.5. Read c off the axis, and take m off a triangle drawn between two crossings of the grid.

Worked example 2 · Read the line

1	Find where the line cuts the y-axis and read the y-value straight off.	**c = 6**
2	Pick two crossings the line passes exactly through: (0, 6) and (4, 0).	**both are exact**
3	Draw the triangle and count the run.	**4 along**
4	Count the rise. The line is falling, so the rise is negative.	**−6**
5	**Calculate** m, then check the sign against the picture.	**m = −1.5, and it falls**
6	**Write down** the equation.	**y = −1.5x + 6**

c is read, not calculated: it is the height at which the line crosses the y-axis. m is calculated from a triangle you draw yourself, and the one thing that decides whether it is right is whether the sign matches a line you can see.

What you want	Where you look for it	What you do with it
the intercept c	where the line cuts the y-axis	read the y-value off the axis — no arithmetic at all
the gradient m	a triangle drawn between two crossings the line passes exactly through	count the run along, count the rise up, and divide
the sign of m	the direction the line travels as you read left to right	climbing is positive, falling is negative
the equation	both of the above, put together	write y = mx + c with your two numbers in it
a check	any third crossing the line passes through	put its x into your equation and see whether its y comes back

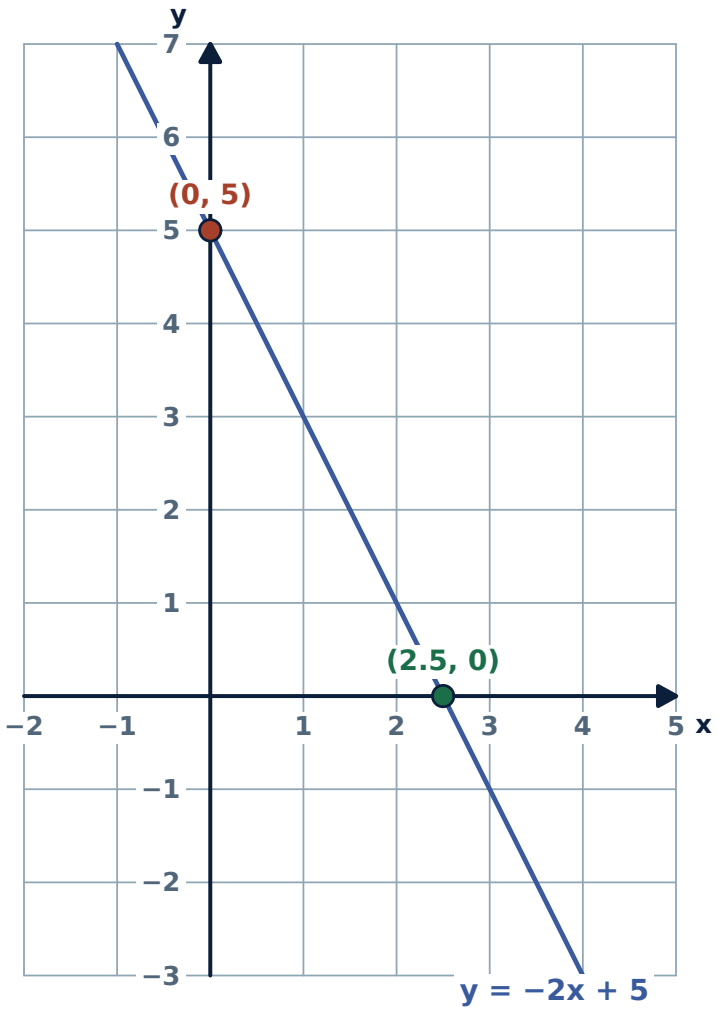
EXAMPLE A line crosses the y-axis at (0, 6), so c = 6. Between (0, 6) and (4, 0) it runs 4 and falls 6, so m = −1.5 and the line is **y = −1.5x + 6**. The picture falls and m is negative, so the two agree.

Concept 3 · Drawing the line from its equation

Two crossings, a ruler and a third point as the check

Going the other way, the two easiest points to find on a line are the two places it crosses the axes, because each of them comes from putting a zero into the equation. Set **x = 0** and the equation hands you c, so the line passes through (0, 5) for y = −2x + 5. Set **y = 0** and solve for x, and you have the other crossing. Two points fix a straight line; a third point, worked out the same way, is what tells you whether the first two were right.

Two crossings are enough to draw a line



Put x = 0 to find where the line crosses the y-axis, and y = 0 to find where it crosses the x-axis. Two points fix a straight line, and a third one worked out the same way is the check that neither of the first two was wrong.

Worked example 3 · Draw it

1	Draw y = −2x + 5. Put x = 0 to find where it cuts the y-axis.	**(0, 5)**
2	Put y = 0 and solve for x to find where it cuts the x-axis.	**(2.5, 0)**
3	Plot both crossings. One of them is not at a grid corner, so mark it halfway between two lines.	**2.5 is halfway**
4	Rule the line through them and let it run past both ends of the grid.	**a line, not a segment**
5	**Verify** it with a third point: put x = 1 into the equation.	**(1, 3)**
6	**Explain** what it means if the third point misses your line.	**one of the three is wrong**

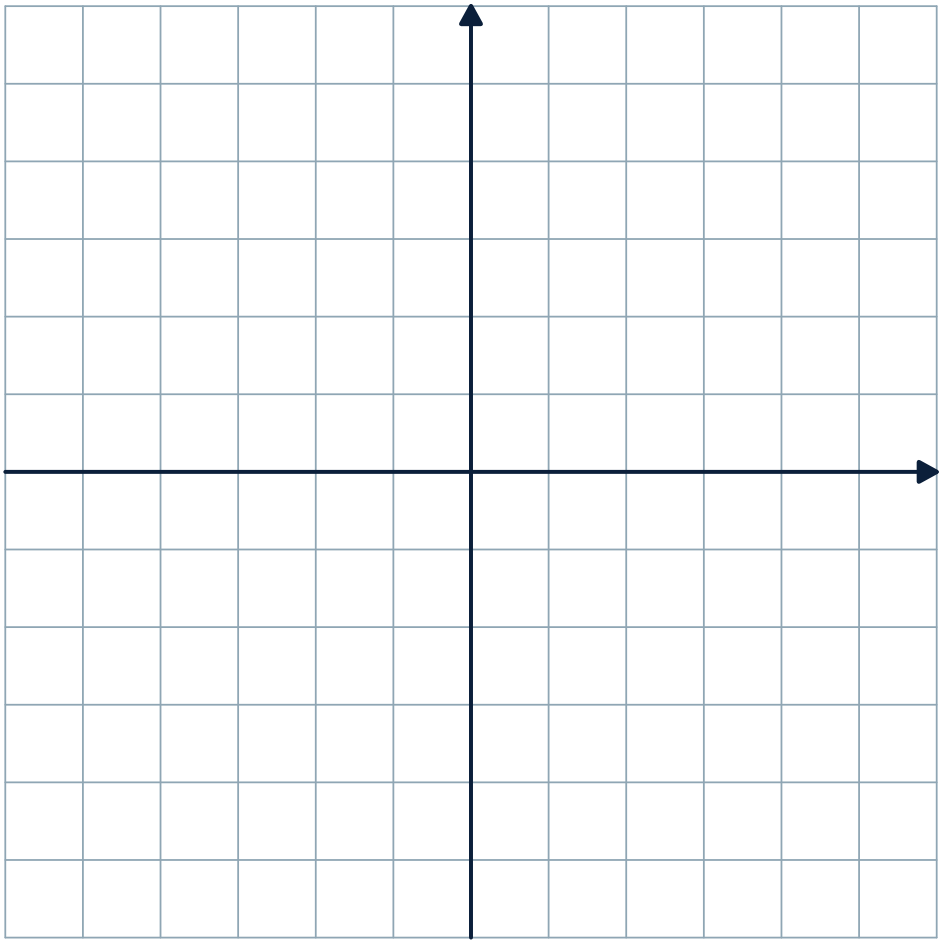
Two points fix a straight line and a third one keeps you honest. The crossings are the two easiest points to find, because each of them comes from putting a zero into the equation — and the awkward one, the one that is not a whole number, is where the mistakes live.

The equation	Crosses the y-axis at	Crosses the x-axis at	One more point
y = −2x + 5	(0, 5)	(2.5, 0)	(1, 3)
y = 3x − 6	(0, −6)	(2, 0)	(1, −3)
y = 0.5x + 1	(0, 1)	(−2, 0)	(4, 3)
y = −x + 4	(0, 4)	(4, 0)	(1, 3)
y = 3	(0, 3)	it never does — it is parallel to the x-axis	(5, 3)

Now draw one yourself

Draw the line y = −x + 4 on the blank axes below, and then y = −2x + 5 on a second set of your own. Number both axes first, work out the two crossings before you pick up a ruler, and mark a third point as the check.

Blank axes, for the drawing questions

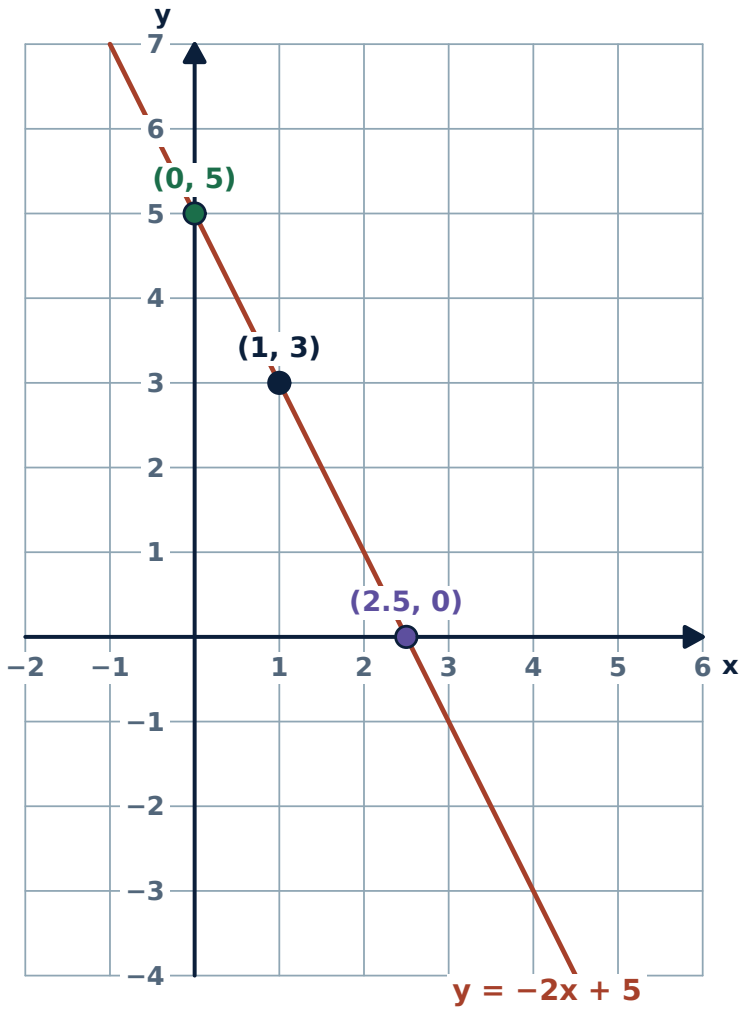


Number both axes from −6 to 6 before you plot anything.

Number the axes yourself, one square to one unit each way. Work out the two crossings first, mark them, and only then put a ruler on the page.

The model diagram for y = −2x + 5 is below. Do not look at it until you have drawn your own.

The model answer, with the checking point on it



The two crossings drew the line and the third point checked it. If your third point misses the line you have drawn, one of the three is wrong — and it is nearly always the crossing with the awkward number in it.

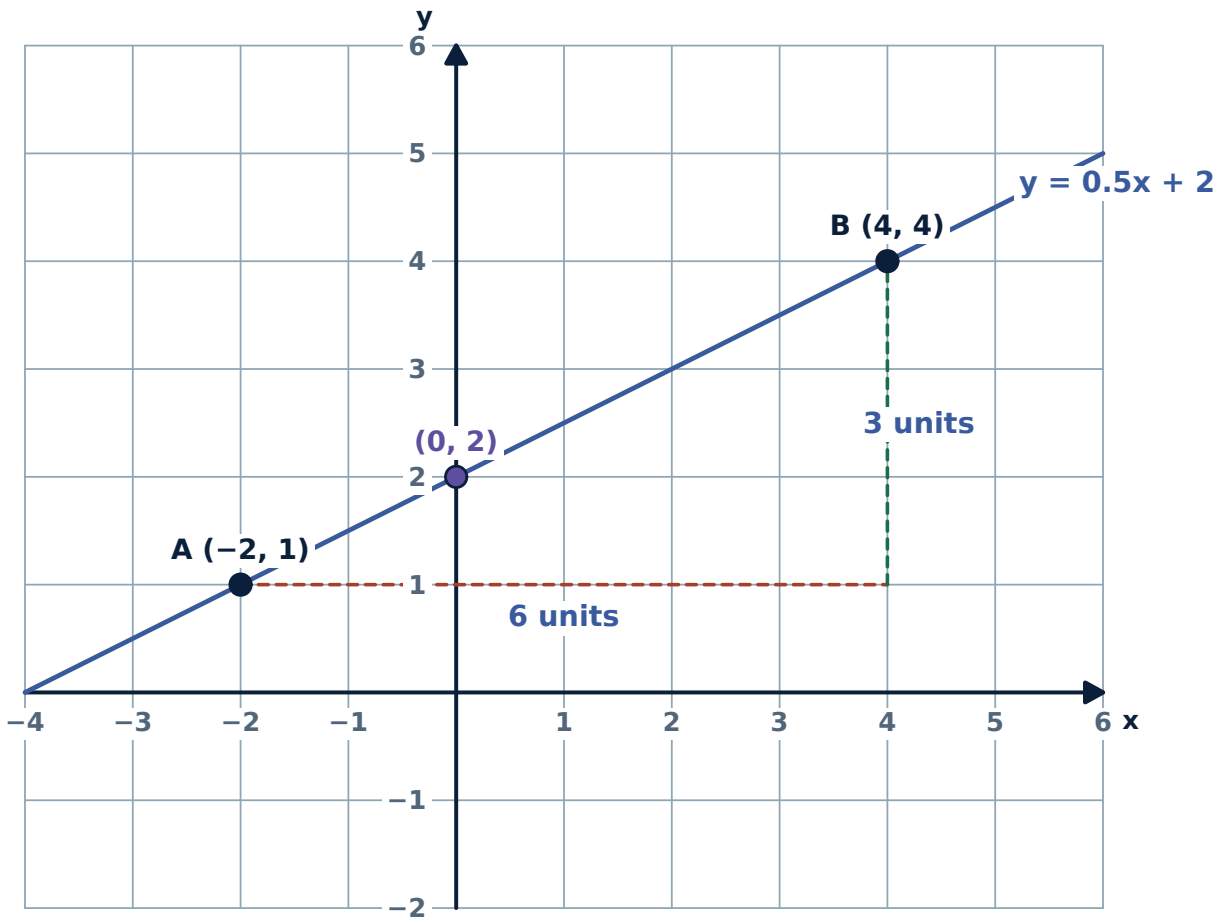
EXAMPLE To draw y = −2x + 5: at x = 0, y = 5, giving **(0, 5)**; at y = 0, x = 2.5, giving **(2.5, 0)**. Rule the line through the two, then check with x = 1: the equation gives y = 3, and (1, 3) should be on your line.

Concept 4 · From two points to an equation

Counting the run and the rise on the grid, and the two lines whose equations look nothing like y = mx + c

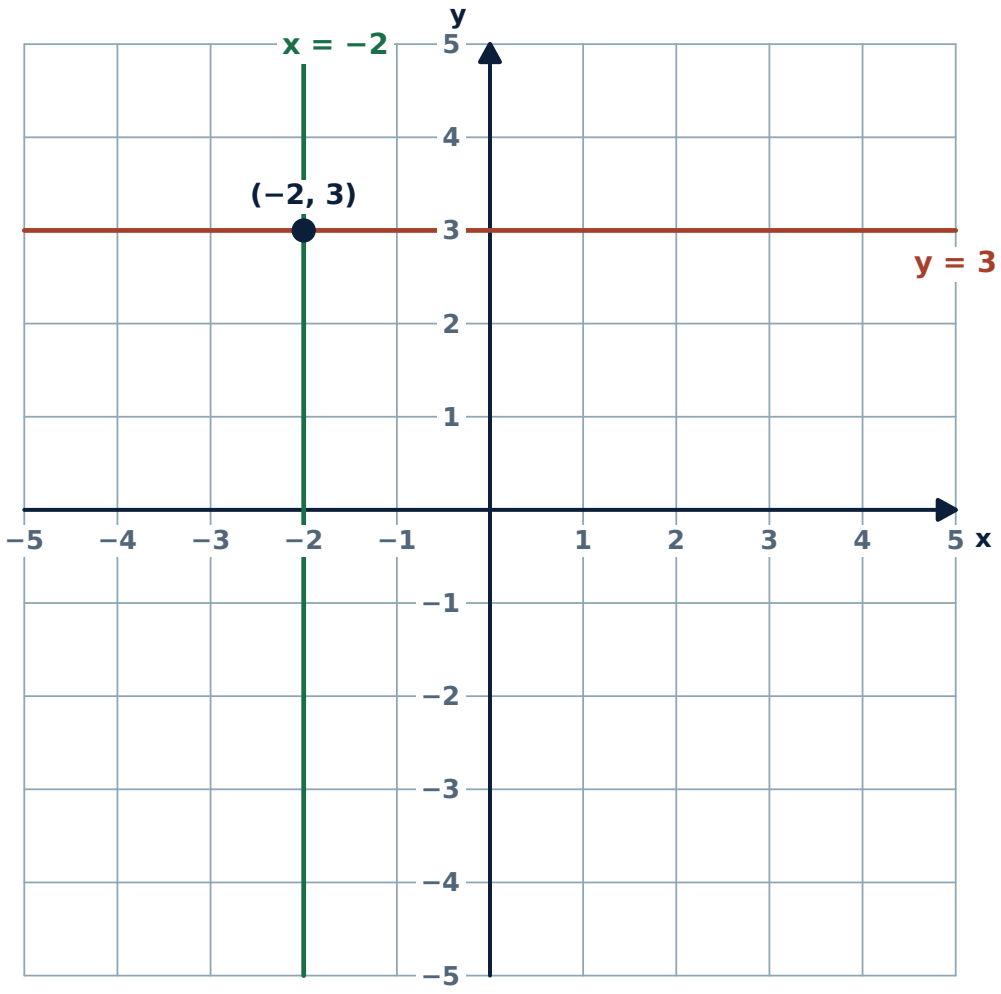
Give me any two points on a line and its equation is **fixed** — there is nothing to guess. The run and the rise between them give m, and then c = y − mx, worked out at either point, gives c; doing it at both points is the check. Two lines break the pattern. A horizontal line has the same y everywhere, so its equation just says so: y = 3. A vertical line has the same x everywhere, so its equation is x = −2 — and because it has no gradient it cannot be squeezed into y = mx + c at all.

Two points, one line, one equation



From A to B the run is 6 and the rise is 3, so the gradient is a half. The line crosses the y-axis at 2, which can be read off the picture here and worked out from either point when it cannot.

The two lines that break the pattern



Every point on the flat line has a y-coordinate of 3, so its equation is y = 3 and its gradient is 0. Every point on the upright line has an x-coordinate of −2, so its equation is x = −2 — and it cannot be written as y = mx + c at all.

Worked example 4 · Two points to an equation

1	Plot A(−2, 1) and B(4, 4) and join them.	**one straight line**
2	Count the run from A to B.	**6**
3	Count the rise from A to B.	**3**
4	**Calculate** m.	**m = 0.5**
5	Use one of the points to find c: c = y − mx, taken at A.	**c = 2**
6	**Verify** with the other point, then **write down** the equation.	**y = 0.5x + 2**

The gradient comes from both points and the intercept comes from either one of them — which is why doing the last step twice, once with each point, catches almost every slip. On this grid c can also simply be read off the y-axis, and the two answers must agree.

The two points	Run	Rise	The equation
(−2, 1) and (4, 4)	6	3	y = 0.5x + 2
(0, −3) and (2, 1)	2	4	y = 2x − 3
(1, 5) and (4, 14)	3	9	y = 3x + 2
(−3, 8) and (3, −4)	6	−12	y = −2x + 2
(3, −4) and (3, 6)	0	10	x = 3 — no m, no c

EXAMPLE Through A(−2, 1) and B(4, 4): the run is 6 and the rise is 3, so m = 0.5. Then c = y − mx at A gives 2, so the line is **y = 0.5x + 2**. B checks it. The vertical line through (3, −4) is **x = 3**, and the horizontal one through the same point is **y = −4**.

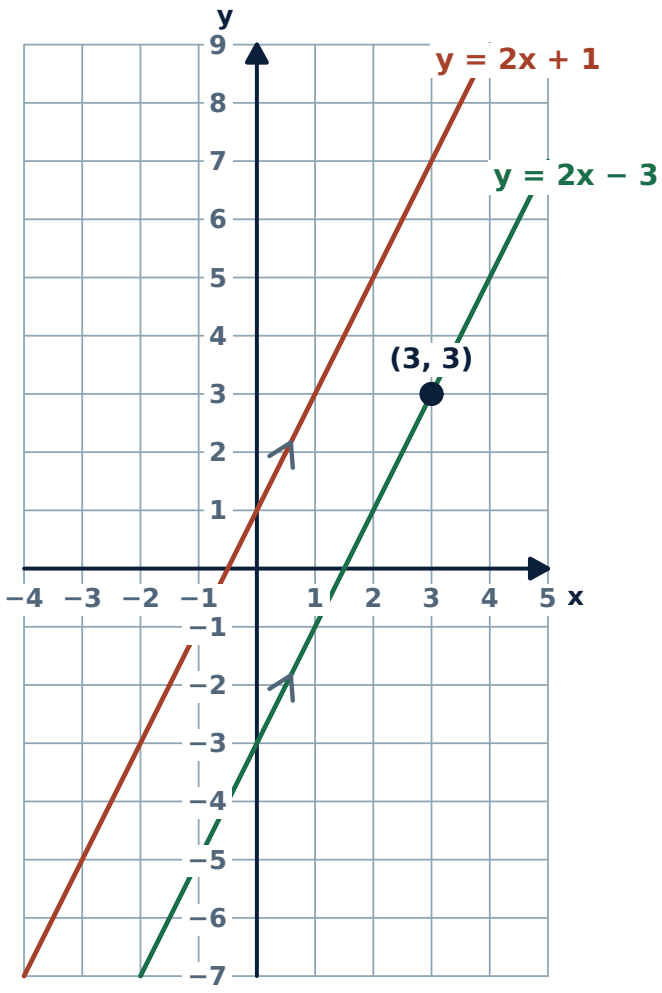
Concept 5 · Parallel lines have equal gradients

Same m, new c — writing the line parallel to a given one through a given point

Changing c slides a line up or down the page without turning it, so every line with the same m points in the same direction and no two of them ever meet: **parallel lines have equal gradients**, and lines with equal

gradients are parallel. To write the line parallel to a given one through a given point, keep m exactly as it is, substitute the point into y = mx + c, and solve the one-step equation for c. Only one line with that gradient goes through that point, and that is the one you have just found.

Same gradient, different intercept



Both lines climb 2 units for every 1 across, so the arrowheads on them are the same and they never meet. Only the intercept was changed, and changing it slid the line up or down without turning it.

Worked example 5 · Parallel through a point

1	The line y = 2x + 1 is given. **State** its gradient.	**m = 2**
2	Parallel means the same steepness, so the new line has the same m.	**m stays 2**
3	The new line must pass through (3, 3). Substitute that point into y = mx + c.	**3 = 2 × 3 + c**
4	**Determine** c.	**c = −3**
5	**Write down** the equation of the parallel line.	**y = 2x − 3**
6	**Explain** how you know the two lines never meet.	**equal gradients**

Changing c slides a line up or down without turning it, so every line with the same m is parallel to every other. Only one of them goes through the point you were given, and substituting the point is what picks it out.

The line you are given	The point it must pass through	The parallel line
y = 2x + 1	(1, −1)	y = 2x − 3
y = −3x + 4	(2, 1)	y = −3x + 7
y = 0.5x − 2	(−4, 3)	y = 0.5x + 5
y = −2x + 7	(3, 0)	y = −2x + 6

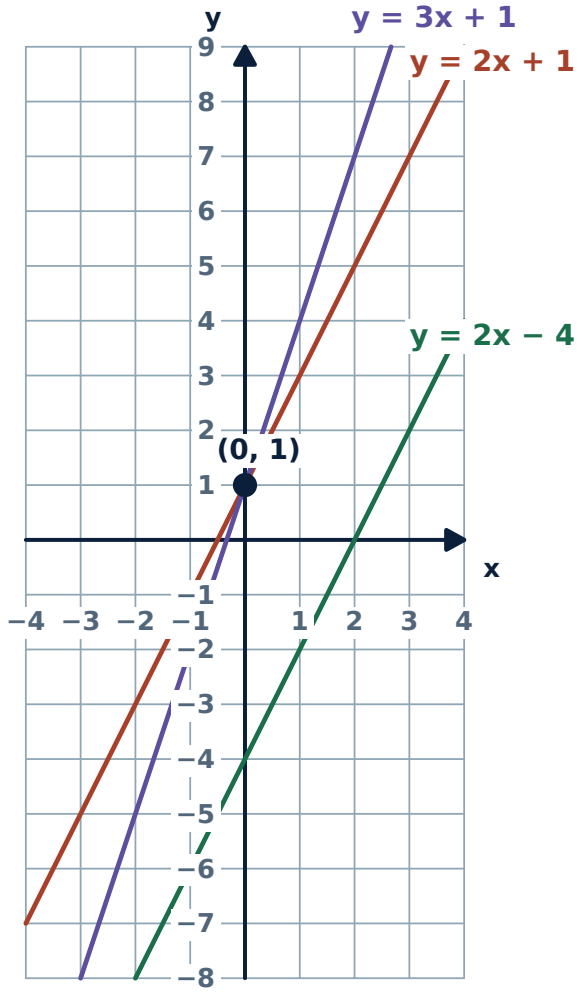
EXAMPLE The line parallel to y = 2x + 1 through (3, 3) keeps m = 2. Substituting the point: 3 = 2 × 3 + c, so c = −3 and the line is **y = 2x − 3**. The intercept changed and the gradient did not, which is exactly what parallel means.

Concept 6 · Deciding which lines are parallel

Four equations, some of them dressed up, and the c that fools people

Deciding which of several lines are parallel is a two-stage job, and the first stage is the one people skip. **Get every equation into y = mx + c first.** 2y = 8x + 6 is not in that form until you halve it; y = 4 − x is written back to front and its gradient is −1, not 4. Only then compare the **gradients**, and only the gradients: two lines that share a c are guaranteed to cross on the y-axis, which is the opposite of parallel, and two lines that share both m and c are one line written twice.

Sharing a c is not being parallel



Two of these three lines are parallel and it is not the two that meet the y-axis at the same place. The gradient decides parallel; the intercept decides only where the line sits.

Worked example 6 · Which of these are parallel?

1	Four lines: y = 4x − 1, 2y = 8x + 6, y = 4 − x and y = −x + 9. **State** the gradient of the first.	**4**
2	The second is not in the standard form. Halve both sides — that is Algebra u18 *The Straight Line*'s move, borrowed.	**y = 4x + 3**
3	The third has its terms the other way round. Reading the x-term gives the gradient.	**−1**
4	The fourth is already in the standard form.	**−1**
5	**Identify** the parallel pairs by their gradients.	**two pairs**
6	**Explain** why the first and the third are not parallel even though both look tidy.	**4 is not −1**

Only the gradient decides. Before you compare anything, get every equation into y = mx + c, and remember that a line whose equation is written back to front, like y = 4 − x, has a gradient of −1 and not 4.

As it is written	Tidied into y = mx + c	Gradient	Parallel to
y = 4x − 1	y = 4x − 1	4	the second row
2y = 8x + 6	y = 4x + 3	4	the first row
y = 4 − x	y = −x + 4	−1	the fourth row
y = −x + 9	y = −x + 9	−1	the third row
y = x + 4	y = x + 4	1	none of the others

EXAMPLE y = 2x + 1 and y = 3x + 1 both cross the y-axis at 1 and are **not** parallel: their gradients are 2 and 3. y = 2x + 1 and y = 2x − 4 share nothing on the page except the 2 in front of the x, and they are **the parallel pair**. Compare gradients, never intercepts.

The gradient routine

1. Pick two points the line goes exactly through — Not two points near it — two crossings of the grid that the line passes through cleanly. Two that are far apart give a more reliable reading than two that are close together.

2. Count the run, then the rise, and keep the signs — Along first, then up: the same order as a coordinate pair. Going right makes the run positive; going up makes the rise positive. Going down makes the rise *negative*, and that minus sign is the answer to half the marks in this topic.

3. Divide, and check the sign against the picture — m = rise ÷ run. Then look at the line: if it climbs from left to right your m must be positive, and if it falls your m must be negative. If the two disagree, trust the picture and find the lost sign.

✓ **Read c off the y-axis, then test one more point** — c is the y-value where the line crosses the y-axis — read it straight off. Then put the x-coordinate of some other point on the line into y = mx + c and check that the y-coordinate comes back.

The mistakes that cost the most marks

Trap 1 · Reading a gradient off a graph whose axes are drawn to different scales

WRONG “The line looks about 60 degrees to me, so the gradient is steep — call it 3.” The gradient has been guessed from how the line looks on the page rather than counted off the two axes.

RIGHT **Count, never measure.** A gradient is a number of vertical units for every horizontal unit, and the only place those units are recorded is on the axes. In the figure below the grid squares are two and a half times taller than they are wide, so the line y = x — whose gradient is exactly 1 — looks far steeper than it is. Count the run and the rise in *units* and the answer comes out right whatever the drawing has done.

Real graphs almost never have matched scales: distance in hundreds of kilometres against fuel in litres, time in seconds against height in metres. On a graph like that the gradient is not a slope you can see — it is a rate, and it carries the unit of the vertical axis over the unit of the horizontal one. A protractor is of no use to you here, and the sign that a diagram is deliberately testing this is the words “Diagram not to scale” printed under it.

The same gradient, drawn on a squashed grid

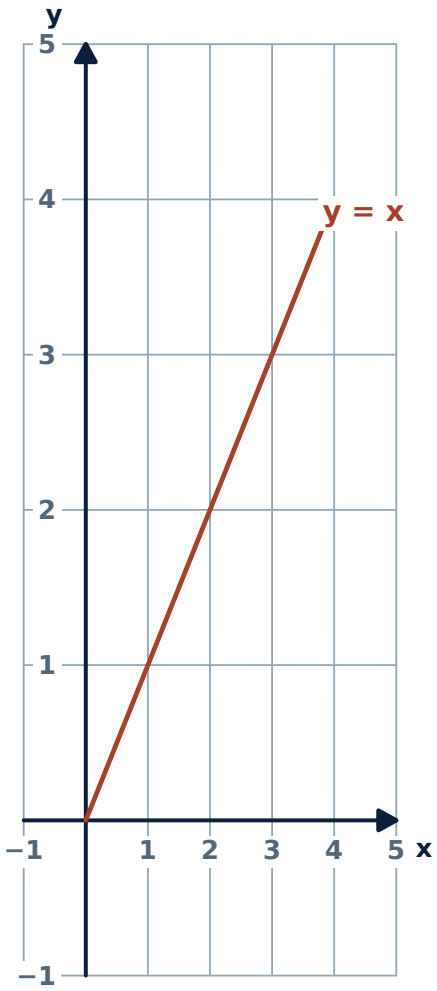


Diagram not to scale

This is the line y = x, whose gradient is exactly 1, drawn on a grid whose squares are two and a half times taller than they are wide. Measured with a protractor it looks far steeper than 1. Counted off the axes it is still 1.

Trap 2 · Losing the minus on a falling line

WRONG “It goes 4 along and 6 down, so the gradient is 6 over 4, which is 1.5.” The size is right and the sign has been dropped, because the fall was counted as though it were a rise.

RIGHT A fall is a **negative rise**. From (0, 6) to (4, 0) the run is 4 and the rise is −6, so m is −1.5. Doing the subtractions in the other order gives the same answer, because both the run and the rise change sign at once: that is the check.

The two-second test that catches every version of this: look at the line and ask whether it climbs or falls as your eye travels left to right. Climbing means m is positive, falling means m is negative. If your arithmetic disagrees with the picture, the picture is right. The commonest cause is subtracting the coordinates in one order for the rise and the other order for the run, which flips one sign and not the other.

Trap 3 · Calling two lines parallel because their c values match

WRONG “y = 2x + 1 and y = 3x + 1 both cross the y-axis at 1, so they are parallel.” The wrong one of the two numbers has been compared.

RIGHT **Parallel is decided by m alone.** y = 2x + 1 and y = 3x + 1 have gradients 2 and 3, so they are not parallel — they cross, and they cross at (0, 1), the very point their matching c values describe. It is y = 2x + 1 and y = 2x − 4, which share a gradient and nothing else, that never meet.

Two lines with the same c are guaranteed to *meet*, on the y-axis, which is the exact opposite of parallel — so this error turns the answer upside down rather than merely nudging it. And if two lines have the same m *and* the same c they are not parallel either: they are the same line, drawn twice. Compare the gradients first, and only then look at the intercepts to see whether you have two lines or one.

Practice · Fluency · 24 questions

P1	A line rises 6 units for every 3 units it runs. **State** its gradient.
P2	**Calculate** the gradient of the line through (1, 1) and (4, 7).
P3	**Calculate** the gradient of the line through (0, 6) and (4, 0).
P4	**State** the gradient of a horizontal line.
P11	A line crosses the y-axis at (0, 6). **Write down** the value of c.
P12	**State** the gradient of the line drawn through (0, 6) and (4, 0).
P13	**Write down** the equation of that line.
P14	**State** the y-intercept of $y = 4x - 7$.
P21	**Write down** where $y = -2x + 5$ crosses the y-axis.
P22	**Determine** where $y = -2x + 5$ crosses the x-axis.
P23	**Determine** the two crossings of $y = 3x - 6$.
P24	Starting at (0, 4) on the line $y = x + 4$, **write down** the point one step right and one step up.
P31	**Determine** the equation of the line through (−2, 1) and (4, 4).
P32	**Write down** the equation of the horizontal line through (3, −4).
P33	**Write down** the equation of the vertical line through (3, −4).
P34	**Determine** the equation of the line through (0, −3) and (2, 1).
P41	**State** the gradient of any line parallel to $y = 2x + 1$.
P42	**Write down** the equation of the line parallel to $y = 2x + 1$ that crosses the y-axis at (0, −3).
P43	**State** whether $y = 4x + 2$ and $y = 4x - 9$ are parallel.
P44	**State** the gradient of a line parallel to $y = 6 - 3x$.
P51	**Identify** which two of $y = 3x + 1$, $y = 3x - 4$ and $y = 2x + 1$ are parallel.
P52	**State** the gradient of the line $2y = 6x + 8$.

P53 **State** whether y = 5 − x and y = −x + 2 are parallel.

P54 **State** whether y = 2x + 5 and y = 5 + 2x are the same line.

Practice · Applied · 24 questions

P5 **Calculate** the gradient of the line through (−2, −3) and (2, 5), and **describe** the run and the rise you used.

P6 **Calculate** the gradient of the line through (−3, 4) and (5, 0).

P7 **Explain** what it means for a line to have a gradient of 0.

P8 **Explain** why the gradient of a vertical line is undefined.

P15 A line crosses the y-axis at (0, −2) and rises 3 units for every 1 unit it runs. **Determine** its equation.

P16 A line crosses the y-axis at (0, 5) and falls 2 units for every 4 units it runs. **Determine** its equation.

P17 **Describe** how to read the gradient off a line drawn on a grid.

P18 **Determine** the coordinates of the two points where y = 2x + 1 crosses the axes.

P25 **Describe** how to draw y = −2x + 5 on a grid using its two crossings.

P26 **Determine** the points on y = 0.5x + 1 where x is −2, 0 and 4.

P27 **Draw** the line y = −x + 4 on the blank grid printed with this unit, and **state** its two crossings.

P28 **Determine** whether (3, −1) lies on y = −2x + 5.

P35 **Determine** the equation of the line through (1, 5) and (4, 14).

P36 **Determine** the equation of the line through (−3, 8) and (3, −4).

P37 **Explain** why the equation of a vertical line cannot be written in the form y = mx + c.

P38 A line has gradient 1.5 and passes through (2, 1). **Determine** its equation.

P45 **Determine** the equation of the line parallel to y = 2x + 1 that passes through (1, −1).

P46 **Determine** the equation of the line parallel to y = −3x + 4 through (2, 1).

P47 **Determine** the equation of the line parallel to y = 0.5x − 2 through (−4, 3).

P48 **Explain** why two lines with the same gradient can never meet.

P55 Four lines: y = 4x − 1, 2y = 8x + 6, y = 4 − x and y = −x + 9. **Determine** which are parallel to which.

P56 **Explain** why y = 2x + 3 and y = 3x + 2 are not parallel, even though they use the same two numbers.

P57 **Determine** whether the line through (0, 1) and (2, 7) is parallel to y = 3x − 5.

P58 **Determine** whether the line through (−2, 4) and (2, −4) is parallel to y = −2x + 9.

Practice · Challenge · 12 questions

P9 A line has gradient −0.75 and passes through (4, 1). **Determine** the y-coordinate of the point on it whose x-coordinate is 8.

P10 The line through (−1, k) and (3, 9) has gradient 2. **Determine** k.

P19 A line drawn on a grid passes through (−2, 7) and (2, −1). **Determine** its equation.

P20 **Explain** why a gradient read by measuring the angle a line makes on the page can be wrong.

P29 The line y = mx + 3 passes through (4, −5). **Determine** m.

P30 The line y = 3x + c passes through (−2, 1). **Determine** c and the x-intercept.

P39 The line through (a, 3) and (5, 11) has gradient 4. **Determine** a.

P40 **Determine** the equation of the line through (−4, 5) and (6, 0).

P49 A line parallel to $y = -2x + 7$ crosses the x-axis at (3, 0). **Determine** its equation.

P50 The line $y = kx + 1$ is parallel to the line through (0, −2) and (3, 7). **Determine** k.

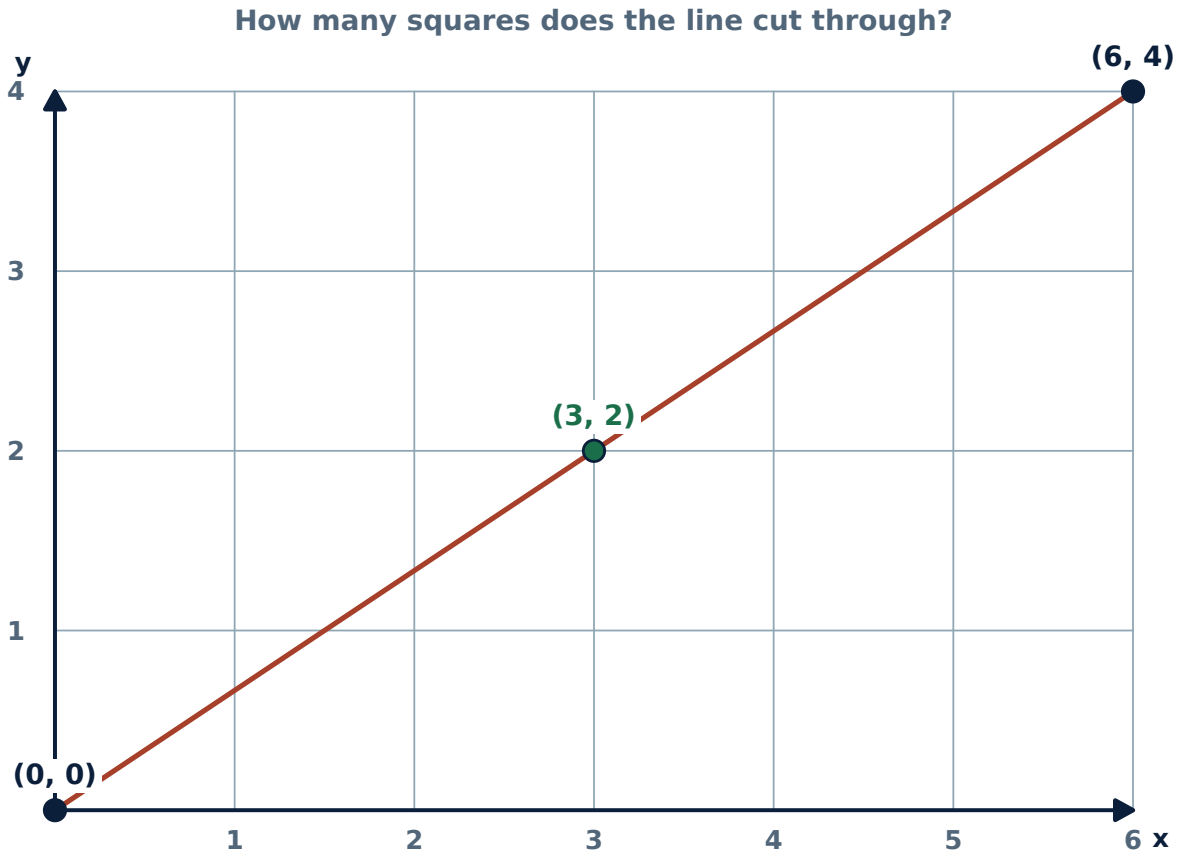
P59 **Determine** the value of c for which $y = 2x + c$ passes through (3, 1), and **explain** how that line is related to $y = 2x - 5$.

P60 Three lines: $y = 2x + 1$, $2y = 4x - 6$ and $y = 2 + x$. **Determine** which are parallel and **explain** the one that is not.

Criterion B · Investigating patterns

How many squares does the line cut through?

Take a sheet of squared paper. Put a ruler on the corner of one square and rule a straight line to the point **a** squares across and **b** squares up. On the way it will pass through the insides of some of the little squares, and the question is **how many**. It is not a formula you have met, it is not in any of the units before this one, and the answer is not a × b, not a + b, and not anything you can guess from two cases. Shade each square as you count it — the easiest way to get this wrong is to count one square twice, and the second easiest is to count a square the line only touches at a corner.



The segment runs from the corner of the paper to the point (6, 4). The green dot is the one place in between where it passes exactly through a corner rather than through the middle of an edge, and that turns out to be the whole story.

Part 1 · structured

a, across	b, up	Squares the line passes through
2	1	2

a, across	b, up	Squares the line passes through
3	2	4
4	2	
5	3	
6	4	

1. **Predict**, before you count anything, how many squares you think the line to (6, 4) passes through, and say whether you expect the answer to depend on a and b separately or only on how big they are. Write the prediction down; you will be asked to compare it with what you find.

2. **Complete** the table for the five pairs given. Rule each segment from the corner (0, 0) to the point (a, b), shade every square whose inside the line passes through, and count the shaded squares. The first two rows are filled in for you.

3. **Identify** the pattern. **Describe** what happens when a and b have a common factor, and **explain** why the pairs (3, 2) and (4, 2) do not follow the same arithmetic even though they look similar. It will help to mark, on each of your diagrams, every point where the line passes exactly through a corner of the grid.

4. **Determine** a general rule for the number of squares in terms of a and b. Write it as a formula, not in words alone, and use it to **predict** the answer for a = 12 and b = 8.

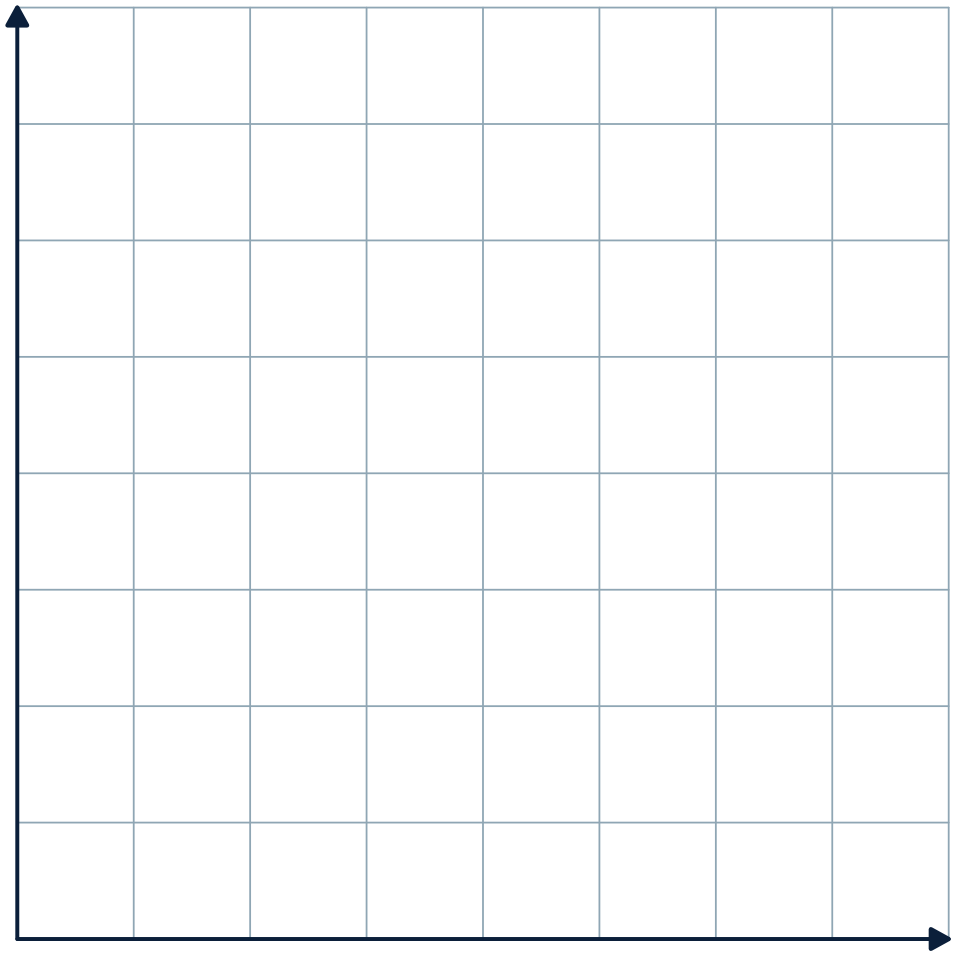
5. **Test and verify** your rule. Check it against a pair you have not already counted, by ruling that one out in full and counting it square by square. Then check it against a = b, where the line runs corner to corner across a square block, and **discuss** whether what your rule says there is right.

6. **Justify** your rule. The line enters a new square every time it crosses a grid line. **Explain** how many vertical grid lines it crosses, how many horizontal ones, and what happens at a point where it crosses one of each at the same instant — and hence why the two counts have to be corrected by exactly the number of such points.

Your own squared paper · the blank grid for steps 2 and 5

Choose a pair of your own, rule the segment from the corner at the bottom left, and shade every square the line passes through. Shading as you go is what stops you counting a square twice. Start at a corner, not in the middle of a square, or the counting will not work.

Your own squared paper



Choose your own a and b, rule the segment, then count the squares.

Pick a pair of your own, rule the segment from the corner, and shade each square as you count it. Shading is what stops you counting a square twice, which is the easiest way to get this wrong.

Part 2 · open

Now you choose what to investigate. Take one of these openings or propose your own. You decide which cases to try, how many you need, how to record what you find and how you will test it.

1. The corners themselves. **Investigate** how many points of the grid the line passes exactly through, not counting the two ends. Tabulate it for several pairs, **determine** a rule, test it, and **explain** how it is related to the rule you already have.

2. Rectangles instead of squares. Suppose the paper is ruled into rectangles two units wide and one unit tall. **Investigate** how many of those the same line passes through, **determine** a rule and **discuss** what changed and why.

3. A line that does not start at a corner. Move the starting point to the middle of a square and rule to (a, b) from there. **Investigate** how the count changes, **determine** what happens when the line no longer passes through any corner at all, and **explain** the connection with your first rule.

Whichever you choose, take it through the same ladder: **predict**, **identify** the pattern, **determine** a general rule, **test and verify** it on a case you have not already counted, and **justify** why it must be true.

Marked out of 8 against Criterion B. Completing the table earns the lower bands; the general rule, the test on a case outside the table and the justification are what reach levels 5–8. Part 2 must show your own choice of method.

Remember, you should communicate in an organized and coherent manner.

Criterion D · Applying maths in real-life contexts

What does the gradient of the fuel line mean?

A small delivery van runs from Gaborone towards Francistown, and its driver keeps a log: at every 80 km she writes down the trip meter and the fuel the van has used since it set off. The log sheet **records a great deal more than you need**. Two questions have to be answered from it. **What does the gradient of the fitted line mean**, in litres for every 100 km — and **is one straight line an honest model of the whole trip**, or is the van doing two different things? Every row of the log is a point you can plot, so both questions are answered by plotting them and looking at the shape they make.

The log sheet (illustrative figures, written for this task)

• **The route:** the first 400 km of a run from Gaborone towards Francistown, logged at every 80 km

• **The van:** a small delivery van with a tank that holds 45 litres

• **The trip meter:** reset to zero at the moment of departure

• **The log:** six readings of distance travelled and fuel used since the start, printed in the table below

• **Departure:** 05:40, before the traffic

• **The load:** two people and about 180 kg of stock

• **Tyre pressure:** checked and set before departure

• **Air conditioning:** switched off for the first three stages and on for the last two

• **Average speed:** about 92 km/h for the first three stages

• **Rest stop:** 25 minutes at the fourth reading

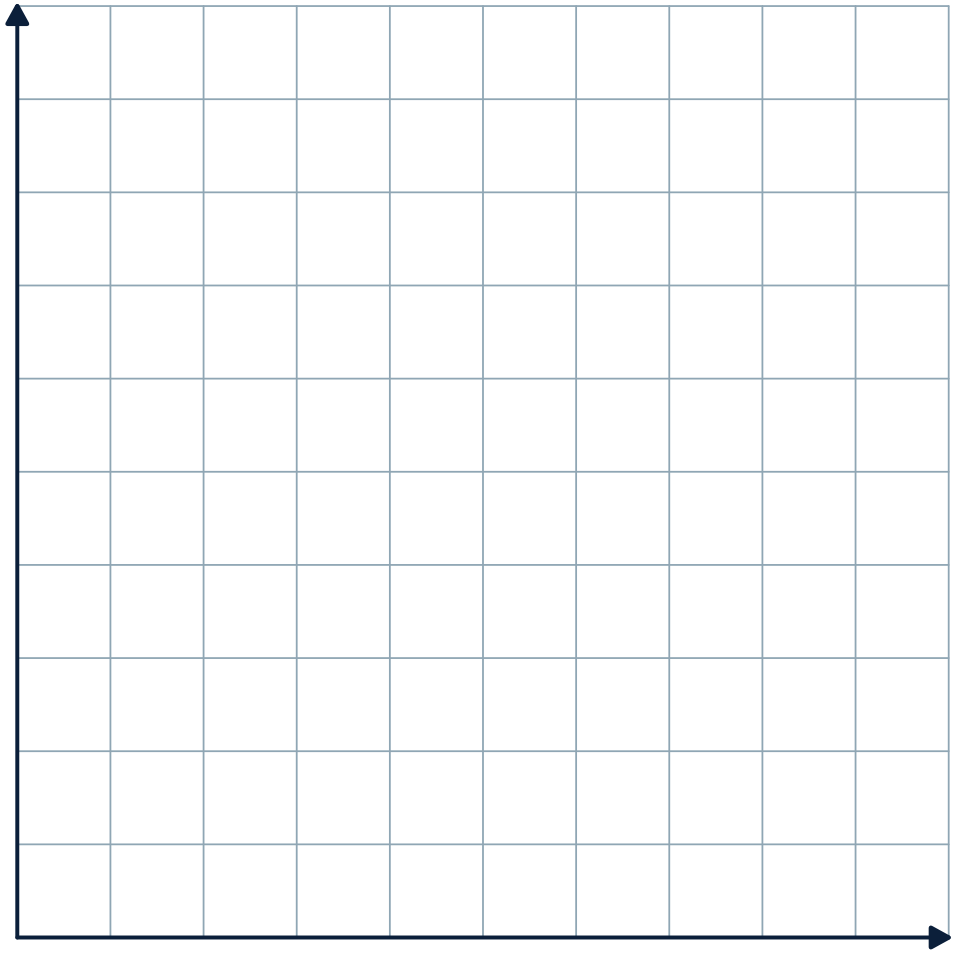
• **The road:** tarred throughout, with roadworks on one stretch

Reading	Distance travelled	Fuel used since the start
1	0 km	0 litres
2	80 km	6.4 litres
3	160 km	12.8 litres

Reading	Distance travelled	Fuel used since the start
4	240 km	19.2 litres
5	320 km	26.4 litres
6	400 km	33.6 litres

These figures are illustrative. The van, the driver and the log sheet are an invented illustrative example written for the task. It is not taken from any business, price list or publication, and no organisation, person or source is named or implied. What is authentic is the shape of the problem: fuel used really does grow in very nearly a straight line with distance, and the rate really does change when the load, the speed or the air conditioning changes.

Blank axes for the fuel log



Choose a scale for each axis before you plot the log.

Distance along the bottom, fuel used up the side. Choose a scale for each axis before you plot a single point, and write the scale on the axis — the two axes here do not have to use the same one, and that is exactly what the first trap in this unit is about.

Identify which entries on the log sheet are needed to answer the two questions and which are not, and **explain**, item by item, why each of the others cannot change either answer — deal with the departure time, the tank size, the tyre pressure, the rest stop and the road surface by name. Then say which single surplus entry you would nevertheless want to know about, and why. *(Strand i — identifying the elements of the authentic situation)*

Select the method you will use. Say which quantity goes on which axis and why that choice makes the gradient mean litres per kilometre rather than kilometres per litre, choose a scale for each axis, and **explain** how you will decide whether one straight line fits the whole log or whether two are needed. *(Strand ii — selecting adequate mathematical strategies)*

Apply them. Plot the six readings on the blank axes. **Calculate** the gradient of the line through the first four readings and the gradient of the line through the last three, convert each into litres per 100 km, and **calculate** the single gradient somebody would get by joining only the first and the last reading. Give all three. *(Strand iii — applying the strategies to reach a solution)*

Explain the degree of accuracy of your answers. The trip meter reads to the nearest kilometre and the fuel gauge is read to the nearest 0.1 litre. **Explain** what that does to a gradient worked out from two readings that are close together, compared with one worked out from two that are far apart, and **explain** how many figures of your answer are worth writing down once you have seen that. *(Strand iv — explaining the degree of accuracy of the solution)*

Discuss whether a single straight line is a reasonable model of the whole trip. Say what the two different gradients would mean to the driver in litres over the rest of the journey, **discuss** what on the log sheet might explain the change, and **discuss** what you would tell her to write in the log next time so that the question could be settled rather than guessed. *(Strand v — explaining whether the solution makes sense in context)*

Submit: your plotted graph with both axes labelled and scaled, your list of which log entries matter and which do not, the three gradients in litres per 100 km, and written answers for strands i, iv and v. **Marked out of 8** against Criterion D at Standard level, where the command terms are **explain** and **discuss**. Give every rate in litres per 100 km and every distance in kilometres.

Answers

How the P-numbers run. The 60 practice items are numbered P1 to P60 in the order they appear in the lesson — ten per concept, and inside each concept four Fluency, then four Applied, then two Challenge. The same P-number identifies the item in the lesson, in this key and in the teacher pack, so the numbers inside each tier below step in blocks of ten rather than running consecutively: Fluency is P1–P4, P11–P14, P21–P24 and so on; Applied is P5–P8, P15–P18, …; Challenge is P9–P10, P19–P20, … A gradient is written as a bare number and carries no unit; a distance carries its unit. Anything that is not exact is given to 3 significant figures.

Fluency · F-tier (24)

P1	**2**
P2	**2**
P3	**−1.5**
P4	**0**
P11	**6**
P12	**−1.5**
P13	**$y = -1.5x + 6$**
P14	**−7**
P21	**(0, 5)**
P22	**(2.5, 0)**
P23	**(0, −6) and (2, 0)**
P24	**(1, 5)**
P31	**$y = 0.5x + 2$**
P32	**$y = -4$**
P33	**$x = 3$**
P34	**$y = 2x - 3$**
P41	**2**

P42	**y = 2x − 3**
P43	**yes — both have gradient 4**
P44	**−3**
P51	**y = 3x + 1 and y = 3x − 4**
P52	**3**
P53	**yes — both have gradient −1**
P54	**yes — they are the same line written two ways**

Applied — fully worked (24)

P5 2; a run of 4 and a rise of 8

The run is the change in x: from −2 to 2 is 4. The rise is the change in y: from −3 to 5 is 8. Dividing, m = 8 ÷ 4 = 2. Counting through zero is where this one goes wrong, so count to the axis and then on: 2 across to the y-axis and 2 more beyond it.

P6 −0.5

The run from −3 to 5 is 8. The line drops from 4 to 0, so the rise is −4. That gives m = −4 ÷ 8 = −0.5. The line falls as you read it left to right, which agrees with the minus sign, and it falls gently, which agrees with the size being less than 1.

P7 it is horizontal — it rises nothing as it runs

A gradient of 0 means the rise is 0 for any run at all, so the line never climbs and never falls: it is horizontal, parallel to the x-axis, and every point on it has the same y-coordinate. It is a perfectly good gradient and a perfectly good line; it is the *vertical* line, with a run of 0, that has no gradient.

P8 its run is 0, and rise divided by 0 is not a number

Take any two points on a vertical line, say (4, −1) and (4, 6). The rise is 7, but the run is 4 − 4 = 0, and the gradient would be 7 ÷ 0. Division by zero has no answer, so the gradient is **undefined** — which is not the same as saying it is zero. A horizontal line has a gradient of 0; a vertical one has none at all.

P15 y = 3x − 2

The intercept is read straight off: c = −2. The gradient is 3 divided by 1, which is 3. Putting the two together gives y = 3x − 2. Check it on a second point: at x = 1 the equation gives y = 1, and going 1 along and 3 up from (0, −2) does land on (1, 1).

P16 y = −0.5x + 5

The fall makes the rise negative: −2 for a run of 4, so m = −2 ÷ 4 = −0.5. The intercept is 5. The equation is y = −0.5x + 5. A gradient between −1 and 0 describes a line that falls gently, which is what “2 down for every 4 across” means.

P17 draw a triangle between two crossings the line passes exactly through, count the run and the rise, and divide

Do not measure anything. Find two points where the line passes exactly through a crossing of the grid — the further apart the better, because a small triangle magnifies any error in reading it. Draw the horizontal run and the vertical rise as the two short sides of a right angled triangle, count both in units off the axes, keep the sign of the rise, and divide the rise by the run.

P18 (0, 1) and (−0.5, 0)

Put x = 0: y = 1, so the line crosses the y-axis at (0, 1). Put y = 0: 0 = 2x + 1, so x = −0.5 and the line crosses the x-axis at (−0.5, 0). Rearranging to find the second one is Algebra u18 *The Straight Line*’s move, used here rather than retaught.

P25 plot (0, 5) and (2.5, 0), rule the line through them, then check a third point

Put x = 0 into the equation: y = 5, giving (0, 5) on the y-axis. Put y = 0: 0 = −2x + 5, so x = 2.5, giving (2.5, 0) on the x-axis, halfway between two grid lines. Rule the line through the two and run it past both ends of the grid. Then test x = 1: the equation says y = 3, and (1, 3) should be on the line you drew.

P26 (−2, 0), (0, 1) and (4, 3)

Substitute each value of x in turn. At x = −2, y = 0.5 × (−2) + 1 = 0. At x = 0, y = 1, which is the intercept. At x = 4, y = 0.5 × 4 + 1 = 3. Choosing even values of x keeps a half gradient tidy; odd ones would land the points between grid lines.

P27 (0, 4) and (4, 0)

Put x = 0 to get (0, 4) and y = 0 to get (4, 0). Both are at grid corners, so this one is quick to plot. Rule the line, then check with x = 1: the equation

gives y = 3 and (1, 3) should sit on it. The gradient is −1, so the line drops one square for every square it moves right — a useful visual check.

P28 yes, it does

Substitute x = 3 into the equation: y = −2 × 3 + 5 = −6 + 5 = −1. That is the y-coordinate the point claims, so (3, −1) is on the line. Testing a point is always this: put its x in, and see whether its y comes back out.

P35 y = 3x + 2

The run from 1 to 4 is 3 and the rise from 5 to 14 is 9, so m = 9 ÷ 3 = 3. Then c = y − mx at (1, 5) gives c = 5 − 3 = 2. The line is y = 3x + 2. Checking with the other point: 3 × 4 + 2 = 14, as required.

P36 y = −2x + 2

The run from −3 to 3 is 6 and the rise from 8 to −4 is −12, so m = −12 ÷ 6 = −2. Then c = 8 − (−2 × (−3)) = 8 − 6 = 2. The line is y = −2x + 2. Both signs in that second step are easy to lose, so check with the other point: −2 × 3 + 2 = −4.

P37 because it has no gradient, and because y is not fixed by x on it

A vertical line has a run of 0, so there is no number m to put in the equation. Worse, the form y = mx + c promises one y for each x, and a vertical line has every possible y sitting above a single x. What is true of it is that the x-coordinate never changes, so its equation says exactly that: x = 3, for the line through (3, −4).

P38 y = 1.5x − 2

Substituting the point into y = mx + c gives 1 = 1.5 × 2 + c, so 1 = 3 + c and c = −2. The line is y = 1.5x − 2. Check it: at x = 2 the equation gives 3 − 2 = 1, which is the point given. A gradient of 1.5 means the line rises 3 units for every 2 across, which is easier to plot than one and a half squares at a time.

P45 y = 2x − 3

Parallel means the gradient is unchanged, so m = 2. Substituting the point into y = 2x + c gives −1 = 2 + c, so c = −3 and the line is y = 2x − 3. Check it: at x = 1 the equation gives 2 − 3 = −1, which is the point. The two lines have different intercepts, so they are genuinely parallel and not the same line.

P46 y = −3x + 7

The gradient stays at −3. Substituting: 1 = −3 × 2 + c, so 1 = −6 + c and c = 7. The line is y = −3x + 7. The sign of the −6 is the whole difficulty

here; a student who writes 1 = 6 + c gets c = −5 and a line that misses the point entirely.

P47 y = 0.5x + 5

m stays at 0.5. Substituting: 3 = 0.5 × (−4) + c, so 3 = −2 + c and c = 5. The line is y = 0.5x + 5. Check: at x = −4 the equation gives −2 + 5 = 3. The two lines are 7 units apart vertically, everywhere along their length, which is another way of saying they are parallel.

P48 at every x they are the same fixed distance apart, so they never touch

Take y = mx + c and y = mx + d with c not equal to d. At any value of x the two heights differ by exactly c − d, which never changes and is never zero, so there is no x at which the two lines have the same y. Geometrically, the two lines climb at the same rate, so the gap between them never closes. Solving them together gives c = d, which is false — that algebraic version of the same argument is Algebra u18 *The Straight Line*'s.

P55 the first two are parallel, and the last two are parallel

Get every one into y = mx + c first. The first already is, with m = 4. Halving 2y = 8x + 6 gives y = 4x + 3, so m = 4 as well: those two are parallel. Reading the x-term of y = 4 − x gives m = −1, and the fourth has m = −1 too: that is the second pair. Four lines, two pairs, and no line is parallel to one from the other pair, because 4 is not −1.

P56 their gradients are 2 and 3, and only the gradient decides

In y = mx + c the two numbers do different jobs: m is the steepness and c is the height at which the line crosses the y-axis. The first line has m = 2 and the second has m = 3, so the second climbs faster and the two must eventually meet. Swapping the numbers between the two roles changes the line completely; it is not a rearrangement of the same line.

P57 yes — both have gradient 3

The run from 0 to 2 is 2 and the rise from 1 to 7 is 6, so the gradient of the first line is 6 ÷ 2 = 3. The second line is already in the standard form and its gradient is 3 as well, so the two are parallel. Their intercepts are 1 and −5, which differ, so they are two lines and not one.

P58 yes — both have gradient −2

The run from −2 to 2 is 4 and the rise from 4 to −4 is −8, so the gradient is −2. The given line also has gradient −2, so they are parallel. Both fall as you read them left to right, which is the picture agreeing with the

arithmetic.

Challenge — fully worked (12)

P9 −2

Going from x = 4 to x = 8 is a run of 4. The gradient says the rise is m × run = −0.75 × 4 = −3, so the y-coordinate falls from 1 to −2. Checking with the equation: c = 1 − (−0.75 × 4) = 4, so the line is y = −0.75x + 4, and putting x = 8 into it gives −2.

P10 k = 1

The run from −1 to 3 is 4, so the rise must be m × run = 2 × 4 = 8. Coming back from 9 by 8 gives k = 1. Check it forwards: the gradient of the line through (−1, 1) and (3, 9) is 8 ÷ 4 = 2, which is what was asked for.

P19 y = −2x + 3

The run from −2 to 2 is 4 and the rise from 7 to −1 is −8, so m = −8 ÷ 4 = −2. Then c = y − mx taken at (2, −1) gives c = −1 − (−2 × 2) = 3. The equation is y = −2x + 3, and the other point checks it: −2 × (−2) + 3 = 7.

P20 because the two axes may be drawn to different scales, so a grid square need not be square

The gradient counts *units*, and the units live on the axes, not on the paper. If one unit across is drawn 1 cm wide and one unit up is drawn 2.5 cm tall, then the line y = x — gradient exactly 1 — will make a much steeper angle on the page than it should. Counting the run and the rise off the axes gives 1 either way. A diagram marked “Diagram not to scale” is telling you this in as many words.

P29 m = −2

Substituting the point gives −5 = m × 4 + 3. Taking the 3 across leaves −8 = 4m, so m = −2. Check it: y = −2x + 3 at x = 4 gives −8 + 3 = −5, which is the point. The line falls, and the point sitting well below the intercept agrees with that.

P30 c = 7; the line crosses the x-axis at (−2.33, 0)

Substituting gives 1 = 3 × (−2) + c, so 1 = −6 + c and c = 7. The line is y = 3x + 7. For the x-intercept put y = 0: 0 = 3x + 7, so x = −7 ÷ 3, which is −2.33 to 3 significant figures. The exact value is the fraction −7/3, and the rounded one is what you would plot.

P39 a = 3

The rise from 3 to 11 is 8. Since m = rise ÷ run, the run must be 8 ÷ 4 = 2. Going back 2 from x = 5 gives a = 3. Check forwards: the gradient through (3, 3) and (5, 11) is 8 ÷ 2 = 4. A student who writes 5 − a = 8 ÷ 4 and then forgets to reverse the subtraction gets a = 7, which fails the check.

P40 y = −0.5x + 3

The run is 10 and the rise is −5, so m = −0.5. Then c = 0 − (−0.5 × 6) = 3, taken at the easier of the two points. The line is y = −0.5x + 3. The other point checks it: −0.5 × (−4) + 3 = 2 + 3 = 5. Note that (6, 0) is the x-intercept, which is why it made the arithmetic shorter.

P49 y = −2x + 6

The gradient is −2, unchanged. The point given is on the x-axis, so substituting it gives 0 = −2 × 3 + c, and c = 6. The line is y = −2x + 6. A point on the x-axis hands you the *x*-intercept rather than the y-intercept, so it still has to be substituted; it cannot be read off as c the way a point on the y-axis can.

P50 k = 3

First find the gradient of the second line: the run is 3 and the rise is 9, so m = 3. Parallel lines have equal gradients, so k = 3. The two lines are then y = 3x + 1 and y = 3x − 2, which have different intercepts and so are two distinct parallel lines rather than one line written twice.

P59 c = −5; it is not merely parallel to y = 2x − 5 — it is the same line

Substituting gives 1 = 6 + c, so c = −5 and the line is y = 2x − 5. That is the line it was to be compared with. Two lines with the same m *and* the same c are one line drawn twice, and one line is not parallel to itself in the sense meant here: parallel lines never meet, and this one meets itself everywhere.

P60 the first two are parallel; the third has gradient 1, not 2

Halving the second gives y = 2x − 3, so its gradient is 2, the same as the first, and their intercepts differ, so they are parallel. The third, written the tidy way round, is y = x + 2: the number multiplying x is 1, not 2. It is the 2 in the constant position that makes it look like a match, and the constant has nothing to do with parallel.