Geometry & Trigonometry · Unit G23 · Extended

Hold More

Extended · 60 practice questions · Criteria A, B, C, D

The twenty-third unit of the Geometry & Trigonometry strand, and the first of the Extended band. A volume becomes a capacity by crossing one exact bridge — and the command term steps up to **justify**.

You already know how to find a volume — **G07** did prisms and cylinders and **G14** did cones. This unit is about the sentence that comes after: *so how much does it hold?* The whole answer rests on one exact fact, **1 cm³ = 1 mL**, and the two thousands built on it. Almost every mark lost here goes in one of three places: a cubic metre divided by a thousand instead of a million, a cone assumed to be half full when the water is half way up, and a capacity quoted to more figures than the tape measure could possibly support.

By the end of this unit you can

• **Convert** between cm³, mL, L and m³ in either direction, and **justify** which power of ten each step needs

• **Calculate** the capacity in litres of a box, a cylinder or a cone from its dimensions, exactly and to 3 significant figures

• **Determine** the capacity held by a part-filled vessel from its depth, and the depth from the capacity, and **explain** why a cone behaves differently from a drum

• **Calculate** filling and draining times from a flow rate in L/min, including a net rate, and read a capacity from a graph

• **Calculate** the capacity of a compound vessel by adding its parts

• **Justify** a stated capacity against the dimensions it is built from, using bounds to decide whether the claim can be defended

The one idea behind all of it

Capacity is volume, wearing a different unit. A vessel's volume is the space its walls enclose and its capacity is what it will hold, and for a

thin-walled container those are the same number crossed over one exact bridge: **1 cm³ = 1 mL**, so 1 000 cm³ = 1 L, and since a metre is 100 cm, **1 m³ = 1 000 000 cm³ = 1 000 L**. Work the volume in whichever unit the measurements come in, keep π exact, and cross the bridge *once*, at the end. Everything else in this unit is a consequence: a part-filled cylinder shares its depth, a part-filled cone shares the *cube* of its depth, a rate problem is a division, and a stated capacity is a claim you are allowed to test.

What you want	The step	In words	Where
A volume in cm³ as a capacity in mL	the same number	1 cm³ is 1 mL — not a conversion	Concept 1
mL as L	÷ 1 000	a thousand millilitres to the litre	Concept 1
cm³ as L	÷ 1 000	because 1 000 cm³ is 1 000 mL	Concept 1
m³ as L	× 1 000	a cubic metre is a thousand litres	Concept 1
m³ as cm³	× 1 000 000	100³, never 1 000 — Trap 1	Concept 1
Capacity of a cylinder	$\pi r^2 h$, then cross once	G07's volume, crossed to litres	Concept 2
Capacity of a cone	$\frac{1}{3}\pi r^2 h$, then cross once	G14's volume, crossed to litres	Concept 2
A part-filled prism or cylinder	the same fraction as the depth	straight sides share the depth	Concept 3
A part-filled cone	the CUBE of the depth fraction	G11's k^3, used and not retaught	Concept 3
Time to fill or empty	capacity ÷ rate	with a net rate when both taps are open	Concept 4
A compound vessel	add the pieces	no join to think about — capacities always add	Concept 5

Work the volume first, in whichever unit the measurements came in, and cross the bridge **once**, at the end. And **say the step out loud** before you write it: if the unit is getting bigger, the number must get smaller.

Keep π exact, and round once, at the end. 72π L is an answer; 226 L is that answer rounded to 3 significant figures. Round, never truncate. And **every answer carries its unit**: cm³ or m³ for a volume, mL or L for a capacity. A bare number is not an answer in this unit, because the whole subject is what the unit does to it.

Where this unit sits

This unit is built on G07, G14 and G11, and adds nothing to any of them. The volume of a prism and of a cylinder came from **G07 Fill the Box**; the volume of a cone and of a compound solid came from **G14 Solid Progress**; and the fact that scaling every length by *k* multiplies a volume by **k³** is **G11 Convert It** and **G18**. None of the three is retaught here. If you are not sure why a cylinder is πr²h, or why a cone is a third of it, spend ten minutes on those units before this one. Choosing a sensible unit in the first place is **N06 Measure It**, and the 3 significant figure rule is **N04 Round It Right**.

Concept 6 borrows one whole argument, and says so. Deciding whether a tank really holds the 5 000 litres printed on its side means asking what its measurements could have been, and that is the upper and lower bound work of **N18 Trapped Between Bounds** in the Number strand's Extended band. This unit uses it and does not reprove it: a length read to the nearest 0.1 m carries a possible error of 0.05 m, and what that does to a capacity is the whole of Strand iv of the Criterion D task.

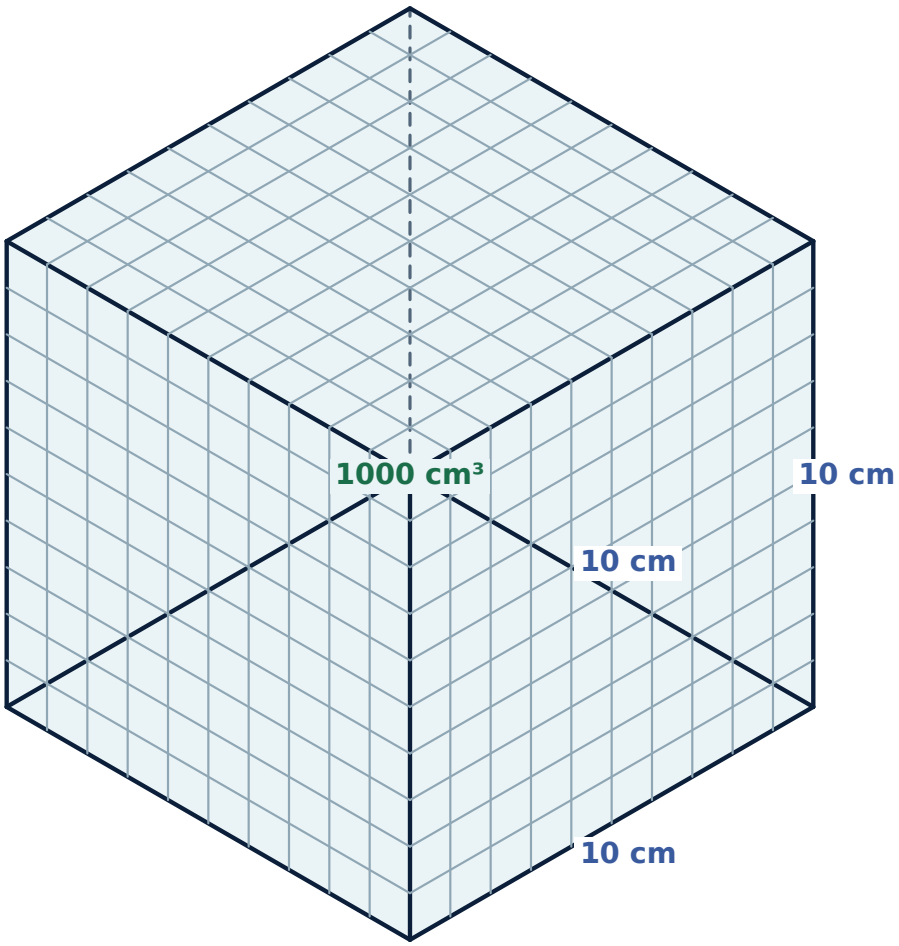
Where this sits in the Extended band. G23 is the first unit of the Extended band, and the command term steps up with it. Foundations asked you to **describe**; Standard asked you to **explain** and **discuss**; from here on the tasks ask you to **justify** — to give a reason that would survive somebody arguing back. In this unit that means never writing a capacity down without being able to say which power of ten took you there and how many of its figures you are entitled to.

Concept 1 · Capacity and volume, and the bridge between them

1 cm³ = 1 mL · 1 000 cm³ = 1 L · 1 m³ = 1 000 L

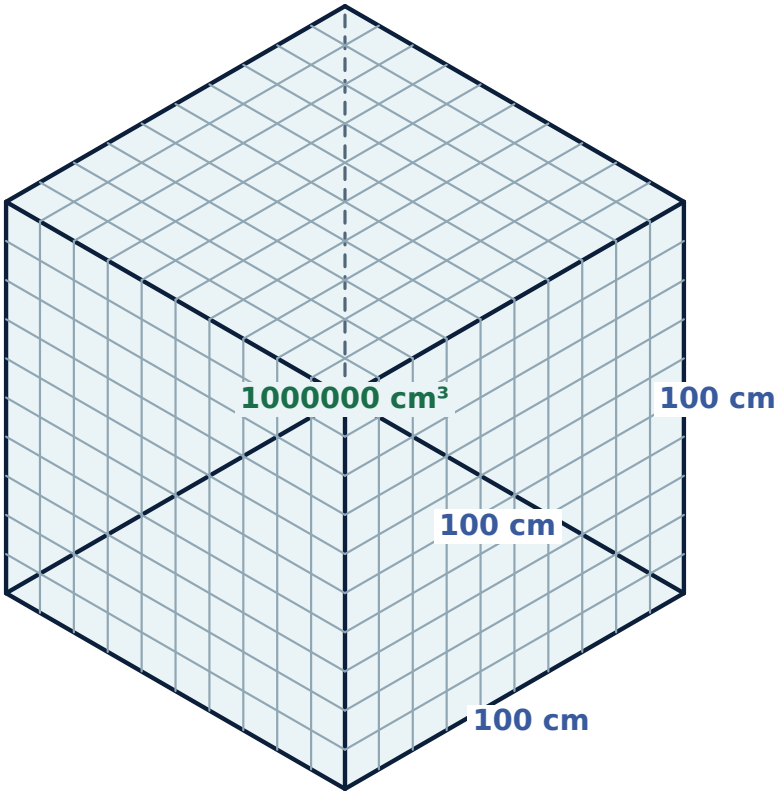
A container's **volume** is the space it encloses and its **capacity** is what it will hold, and for a thin-walled vessel those are the same number wearing different units. The bridge between them is one exact fact: **1 cm³ = 1 mL**. Everything else follows from it and from the prefixes. A thousand millilitres is a litre, so **1 000 cm³ = 1 L**. A metre is 100 centimetres, so a cubic metre is 100^3 = **1 000 000 cm³**, and that is **1 000 L**. Two numbers do all the work — one and a thousand — and a million appears only where two thousands are being crossed at once.

One litre, drawn



A cube 10 cm on every edge holds 10 × 10 × 10 = 1 000 cm³. Every one of the little cubes ruled on its faces is 1 cm³, which is **1 millilitre**, *and 1 000 millilitres is* **1 litre**. *So this box is a litre. That is the whole bridge, and it is exact — there is no rounding anywhere in it.*

One cubic metre is one thousand litres



The same cube with every edge 100 cm — one metre. It is ruled into blocks 10 cm on a side, and each of those blocks is the litre cube above. There are 10 × 10 × 10 = **1 000** *of them, so* **1 m³ = 1 000 L***. In cubic centimetres it is 100³ =* **1 000 000 cm³***, not 1 000 — and that single slip is Trap 1.*

The capacity ladder

the same thing **÷ 1 000** **× 1 000**



Read it left to right and you are going up in size. The first step is not a conversion at all — a cubic centimetre is a millilitre, so nothing changes but the word. The other two steps are both a thousand. Read it right to left and every thousand becomes a division. The one step this ladder does **not** *show is cm³ straight to m³, because that is two steps of a thousand at once: a* **million***.*

Worked example 1 · Across the bridge, in both directions

1	A jug holds 3 500 cm³. **Write down** its capacity in millilitres.	**3 500 mL**
2	**Convert** that to litres.	**3.5 L**
3	A drum is rated 0.85 L. **Convert** that to cubic centimetres.	**850 cm³**
4	A reservoir holds 2.4 m³. **Convert** that to litres.	**2 400 L**
5	**Write down** that same volume in cubic centimetres.	**2 400 000 cm³**
6	Which of those two steps is the one people get wrong, and why?	**the last — a metre is 100 cm, so a cubic metre is 100³ = 1 000 000 cm³**

There are only two numbers in the whole of Concept 1: one (cm³ to mL, which changes nothing) and a thousand (mL to L, and m³ to L). The only place a million appears is between cm³ and m³, and it appears there because that jump is two thousands at once. Say the step out loud before you write it: *am I going up in size or down?* Going up in size means the number gets smaller.

As given	In cm³	In mL	In L	In m³
3500 cm³	3500 cm³	3500 mL	3.5 L	0.0035 m³
0.85 L	850 cm³	850 mL	0.85 L	0.00085 m³
2.4 m³	2400000 cm³	2400000 mL	2400 L	2.4 m³
750 mL	750 cm³	750 mL	0.75 L	0.00075 m³
0.25 L	250 cm³	250 mL	0.25 L	0.00025 m³
12 m³	12000000 cm³	12000000 mL	12000 L	12 m³

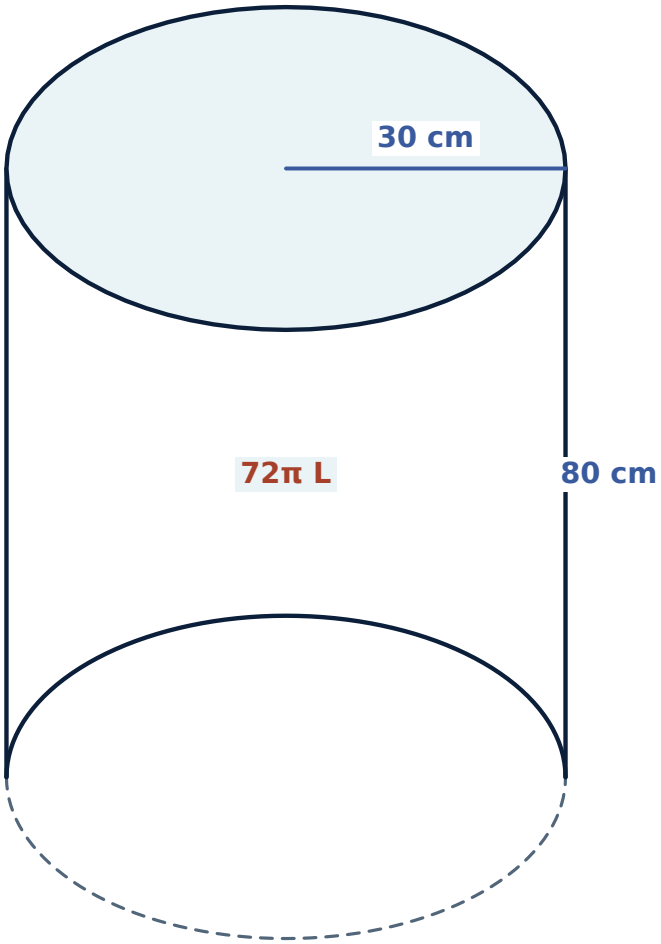
EXAMPLE A jug holds 2.5 L. In millilitres that is 2 500 mL, in cubic centimetres it is **2 500 cm³** — the same number again — and in cubic metres it is 0.0025 m³. Notice that the number gets smaller every time the unit gets bigger, which is the only sanity check the ladder needs.

Concept 2 · Tanks, drums and boxes turned into litres

Work the volume first, then cross the bridge exactly once

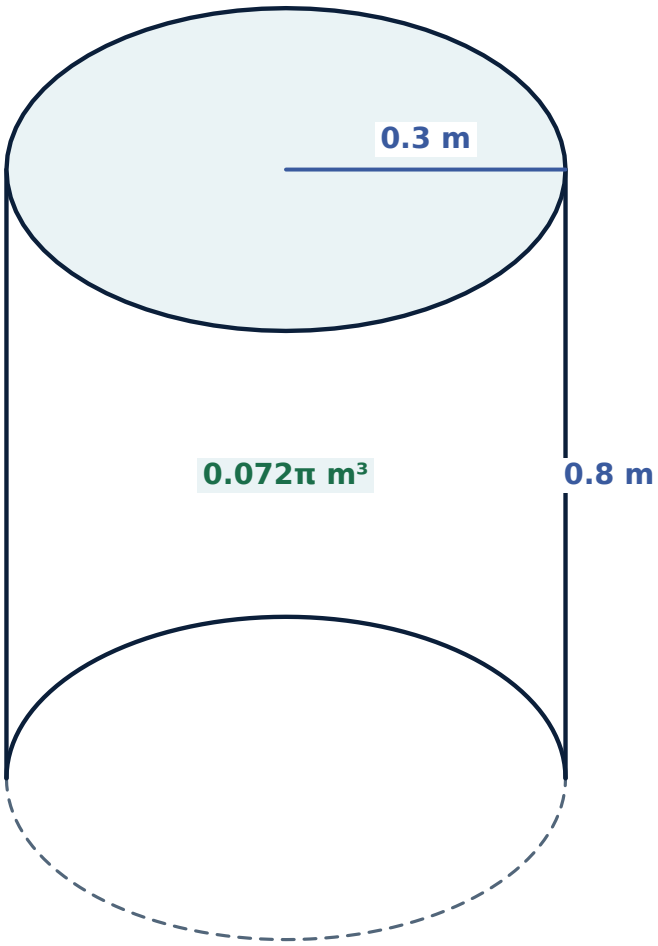
Nothing new is needed to find a capacity: work the **volume** the way **G07** and **G14** taught you, in whichever unit the measurements arrived in, and then cross the bridge *once*. A box is lwh, a cylinder is πr²h, a cone is a third of its cylinder. Two habits pay here and they are the same two as always. Keep π exact until the last line, so a tank is 72π L and only then 226 L. And read the measurement you were given twice, because tanks are quoted by their **diameter** at least as often as by their radius, and halving it is the difference between an answer and four times an answer.

The tank, measured in centimetres



Radius 30 cm and height 80 cm, so the volume is π × 30² × 80 = 72 000π cm³. Cross the bridge once, dividing by 1 000, and the capacity is **72π L**, *which is* **226 L** *to 3 significant figures. Keep the π until the very last line: 226 litres is the rounded answer, 72π litres is the answer.*

The same tank, measured in metres



30 cm is 0.3 m and 80 cm is 0.8 m, so this is the identical tank. Its volume now reads π × 0.3² × 0.8 = **0.072π m³***, which is 0.226 m³ to 3 significant figures. Multiply by 1 000 and the capacity is 72π L again — the same litres, reached from the other side of the bridge. If the two ever disagree, a power of ten has gone missing.*

Worked example 2 · A tank in litres, in cubic centimetres and in cubic metres, all three agreeing

1	A cylindrical tank has radius 30 cm and height 80 cm. **Calculate** its volume, exactly.	**$\pi \times 30^2 \times 80 = 72\,000\pi$ cm³**
2	**Write down** its capacity in millilitres.	**72 000π mL — the same number**
3	**Calculate** its capacity in litres, exactly and to 3 significant figures.	**72π L, which is 226 L**
4	Now measure the same tank in metres. **Write down** its radius and height.	**0.3 m and 0.8 m**
5	**Calculate** its volume in cubic metres.	**$\pi \times 0.3^2 \times 0.8 = 0.072\pi$ m³**
6	**Show that** the two routes agree.	**$0.072\pi \times 1\,000 = 72\pi$ L**

One tank, two sets of numbers, one capacity. That is the check this unit is built on, and it is worth doing in the margin of any question where the units make you uneasy: work it the other way and see whether you land in the same place. Notice how the π travels untouched all the way through, and the rounding happens once, at the very end. 226 L is the rounded answer; 72π L is the answer.

Vessel	Shape	Dimensions	Volume	Capacity	To 3 s.f.
Water tank	cylinder	r = 30 cm, h = 80 cm	72000π cm³	72π L	226 L
Drum	cylinder	r = 25 cm, h = 90 cm	56250π cm³	56.25π L	177 L
Bucket	cylinder	r = 12 cm, h = 35 cm	5040π cm³	5.04π L	15.8 L
Mug	cylinder	r = 4 cm, h = 10 cm	160π cm³	0.16π L	0.503 L
Storage crate	cuboid	30 cm × 20 cm × 25 cm	15000 cm³	15 L	15.0 L
Conical funnel	cone	r = 20 cm, h = 30 cm	4000π cm³	4π L	12.6 L

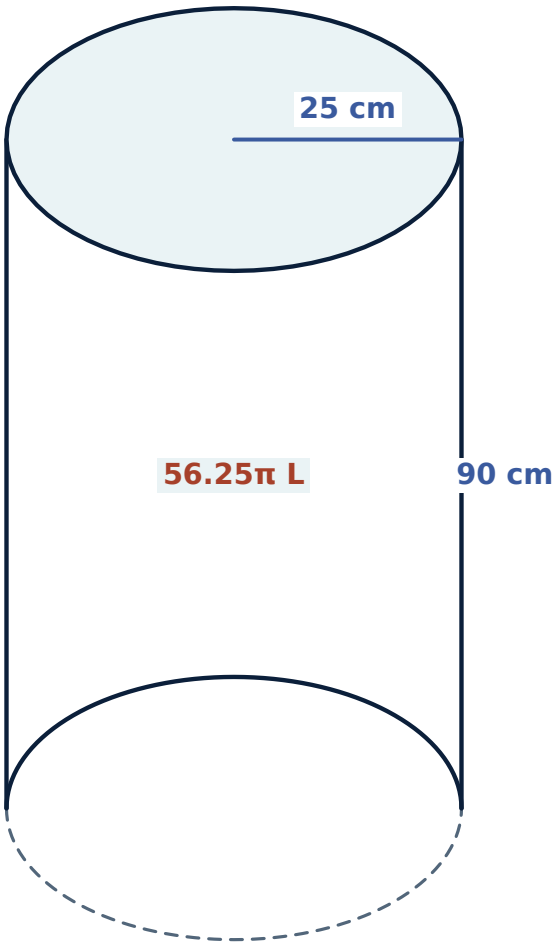
EXAMPLE A cylindrical drum has radius 20 cm and height 60 cm. Its volume is $\pi \times 400 \times 60 = 24\,000\pi$ cm³, so its capacity is **24π L**, which is **75.4 L** to 3 significant figures. Measured in metres the same drum is 0.2 m by 0.6 m and its volume is 0.024π m³ — and $\times\ 1\,000$ brings it back to 24π L, as it must.

Concept 3 · Part full: depth against capacity

Straight sides share the depth; a cone does not

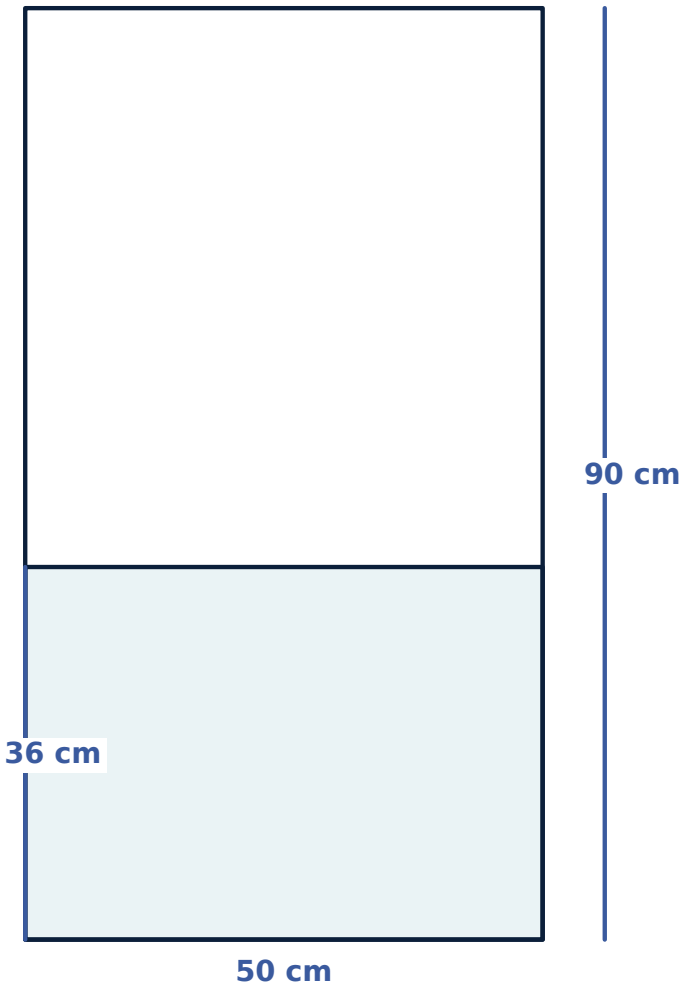
A vessel that is not full holds a smaller vessel's worth of water, and the only question is what shape that smaller vessel is. If the sides are **straight** — a box, a prism, a drum — the water is a shorter copy of the same vessel, so the fraction of the depth and the fraction of the capacity are the **same number**. If the vessel **narrows**, they are not. Water 15 cm deep in a 30 cm cone is a cone of half the depth *and half the radius*, so it holds $(\frac{1}{2})^3$ = **one eighth**. That is the k^3 rule of **G11** and **G18**, and here it changes the answer by a factor of four.

A drum, full to the brim



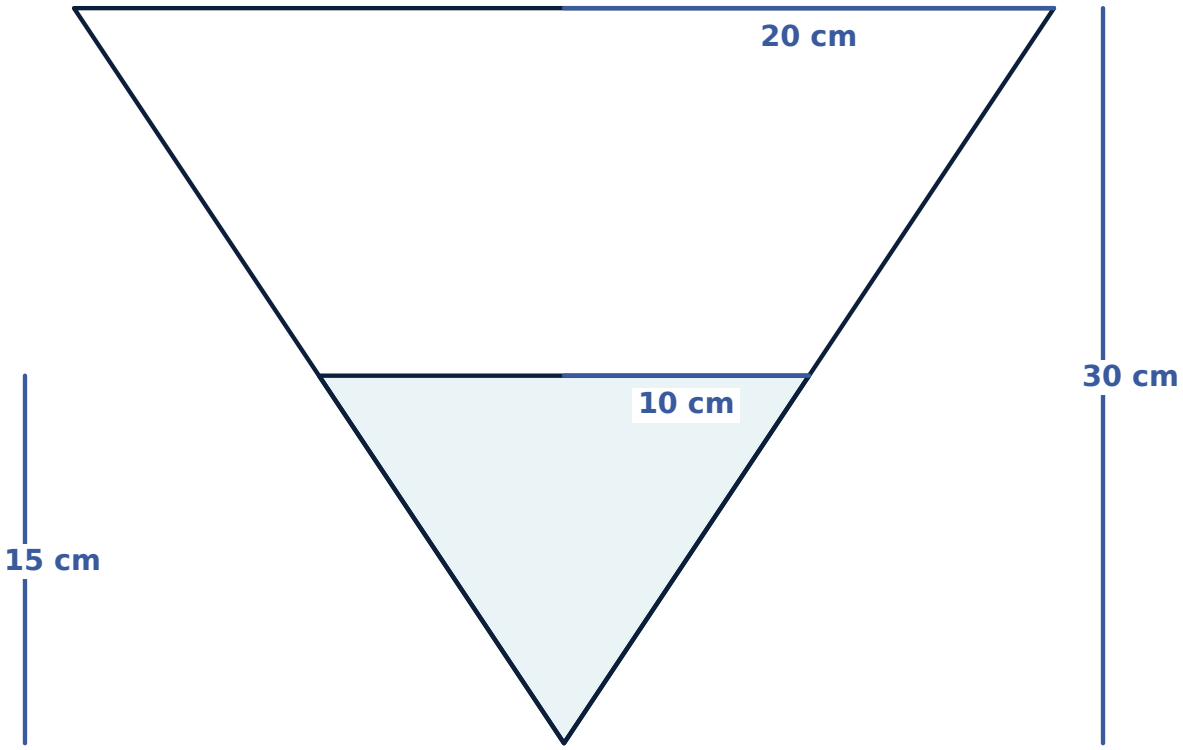
Radius 25 cm, height 90 cm. Full it holds π × 625 × 90 = 56 250π cm³, which is **56.25π L** *or* **177 L** *to 3 significant figures. Everything in this concept is about what happens when it is not full.*

The same drum, cut down the middle



The water is 36 cm deep in a drum 90 cm tall, and because the sides are straight the water is simply a **shorter drum***: π × 625 × 36 = 22 500π cm³ = 22.5π L =* **70.7 L***. The depth fraction and the capacity fraction are the same, 36 ÷ 90 =* **40%***, and that is true of every prism and every cylinder — and of nothing else in this unit.*

A conical vessel, filled to half its depth



A cone standing on its point, 30 cm deep and 20 cm in radius at the rim. Fill it to 15 cm — half the depth — and the water surface is only 10 cm in radius, because the water is a **scaled copy** *of the whole cone with every length halved. Half the depth, half the radius, and therefore* **one eighth** *of the capacity, not one half. That is the cube law of* **G11** *and* **G18***, used and not retaught.*

Worked example 3 · Part full: a drum, and then a cone that refuses to behave

1	A drum of radius 25 cm and height 90 cm holds water 36 cm deep. **Calculate** the capacity it is holding, in litres.	**π × 625 × 36 = 22 500π cm³ = 22.5π L = 70.7 L**
2	**Calculate** the percentage of the drum that is full.	**36 ÷ 90 = 40%**
3	Why is that the same as the depth fraction?	**the sides are straight, so the water is a shorter drum**
4	A conical funnel is 30 cm deep and 20 cm in radius at the rim. It is filled to a depth of 15 cm. **Write down** the radius of the water surface.	**10 cm — half of 20**
5	**Calculate** the capacity it is holding.	**⅓ × π × 100 × 15 = 500π cm³ = 0.5π L = 1.57 L**
6	**Calculate** the percentage of the funnel that is full.	**(15 ÷ 30)³ = 12.5%**

Two vessels, two completely different answers to the same question. A drum half full by depth is half full by capacity; a cone half full by depth is **one eighth** full by capacity, because halving the depth halves the radius as well and the two halvings multiply with a third one. That is the k^3 rule of G11 and G18, arriving in a place where it genuinely changes the answer.

Vessel	Full capacity	Depth	Held	To 3 s.f.	% full
Drum (cylinder)	56.25π L	36 cm	22.5π L	70.7 L	40%
Drum (cylinder)	56.25π L	63 cm	39.375π L	124 L	70%
Water tank (cylinder)	72π L	20 cm	18π L	56.5 L	25%
Funnel (cone)	4π L	15 cm	0.5π L	1.57 L	12.5%
Funnel (cone)	4π L	24 cm	2.048π L	6.43 L	51.2%

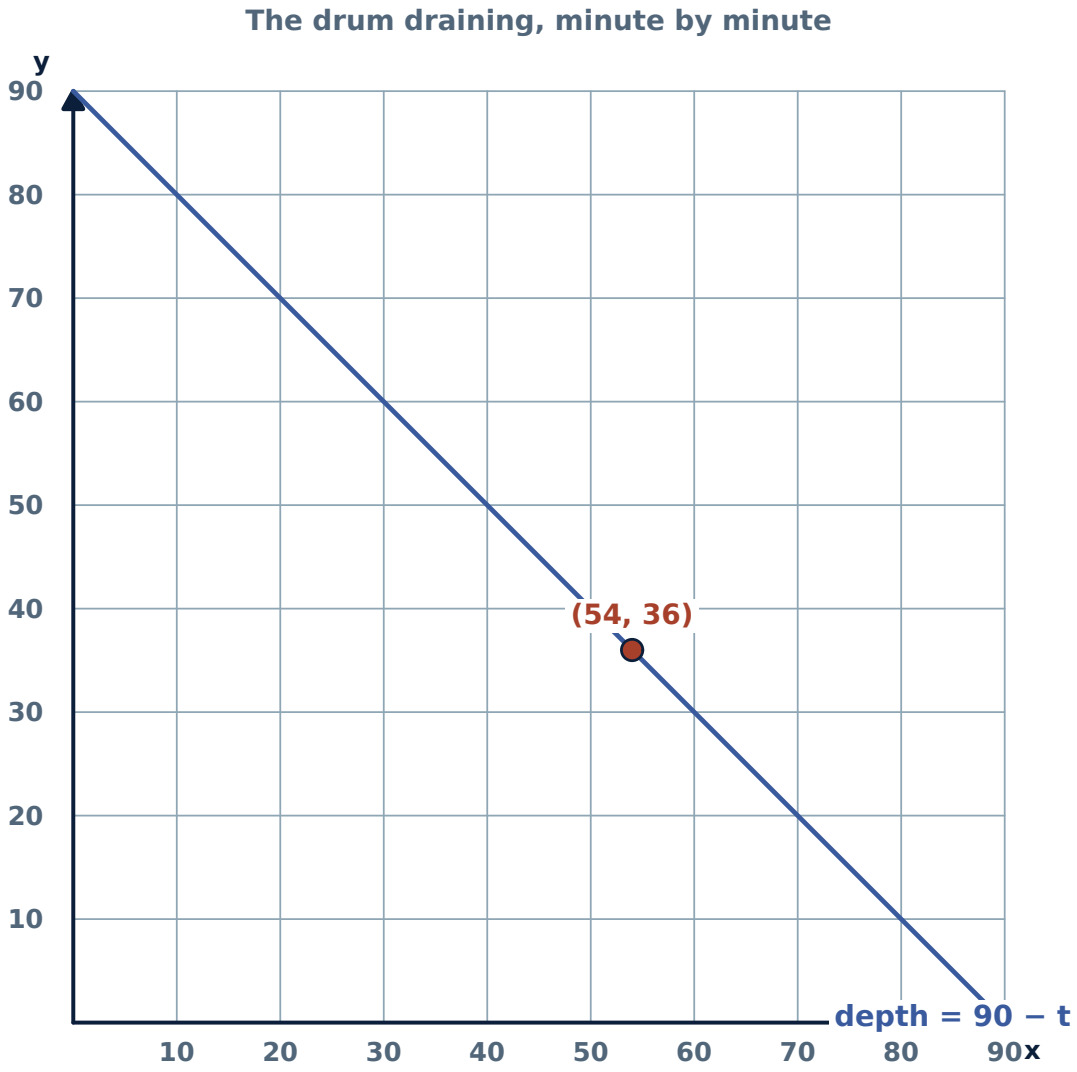
EXAMPLE A drum 60 cm tall filled to 45 cm is 45 ÷ 60 = **75% full**. A cone 60 cm deep filled to 45 cm is (45 ÷ 60)³ = 0.421875, which is

42.2% full. Same vessels, same depths, and a difference of nearly a third of the capacity — which is why the first thing to write down for a cone is the radius of the water surface.

Concept 4 · Rates: filling, draining and how long it takes

Capacity ÷ rate, and the graph that shows it

A rate question is a division and almost nothing else: **time = capacity ÷ rate**. The care goes into the two numbers you divide. Make sure the capacity is in litres if the rate is in litres a minute — a capacity in cm^3 and a rate in L/min will give an answer a thousand times wrong and look perfectly reasonable doing it. And when two taps are open at once, find the **net rate** first and write it on its own line: a leak is a subtraction before the division, never a second division. A steady rate draws a straight line, and the gradient of that line is the rate.



The horizontal axis x is the time in minutes and the vertical axis y is the depth of water in centimetres. The tap at the bottom lets the level fall a steady **1 cm a minute**, *so the graph is a straight line from 90 cm at the start to empty after 90 minutes. Read off 54 minutes and the depth is 36 cm — the drum of the last concept. One centimetre of depth is π × 625 × 1 = 625π cm³, so the tap is running at 0.625π L a minute, about* **1.96 L/min**.

Worked example 4 · How long does it take?

1	The tank of Worked example 2 holds 72π L. A tap fills it at 9 litres a minute. **Write down** the calculation.	**time = capacity ÷ rate = 72π ÷ 9**
2	**Calculate** the time, exactly and to 3 significant figures.	**8π min, which is 25.1 min**
3	A 1 800 L reservoir is filled at 15 L/min. **Calculate** the time.	**120 min**
4	Now the reservoir leaks at 8 L/min while the same tap runs. **Write down** the net rate.	**15 − 8 = 7 L/min**
5	**Calculate** the new filling time.	**1 800 ÷ 7 = 257 min**
6	The drum of Worked example 3 empties at a steady 1 cm of depth a minute. **Calculate** the flow, in litres a minute.	**π × 625 × 1 = 625π cm³ = 0.625π L/min = 1.96 L/min**

Every rate question in this unit is one division, and the only thinking is about which capacity and which rate. When two taps are open at once, work out the **net** rate first and write it down as its own line — a leak is a subtraction, not a second division. And notice the last step: a depth falling at a steady rate in a straight-sided vessel means a steady flow, which is exactly why the graph in this concept is a straight line.

Capacity	In	Out	Net rate	Time	To 3 s.f.
72π L	9 L/min	—	9 L/min	8π min	25.1 min
1800 L	15 L/min	—	15 L/min	120 min	120 min
56.25π L	12 L/min	—	12 L/min	4.6875π min	14.7 min
1800 L	15 L/min	8 L/min	7 L/min	257 min	257 min
900 L	20 L/min	5 L/min	15 L/min	60 min	60.0 min

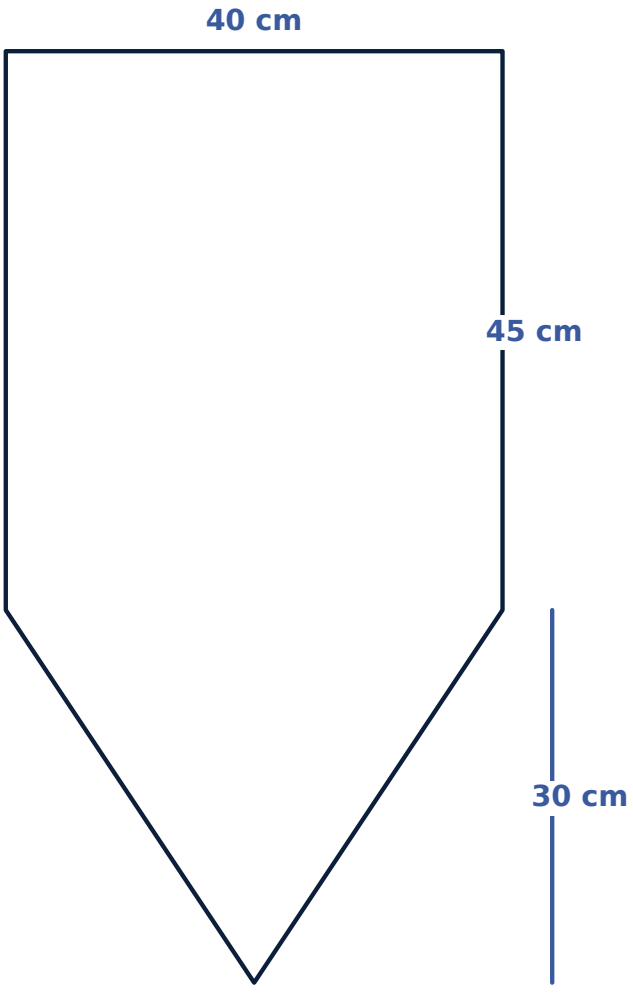
EXAMPLE A 900 L reservoir fills at 20 L/min while draining at 5 L/min. The net rate is 20 − 5 = 15 L/min, so it fills in 900 ÷ 15 = **60 minutes**. Without the leak it would have taken 45 minutes, and over the hour the leak has thrown away 5 × 60 = 300 L — a third of the reservoir.

Concept 5 · Compound vessels

Capacities add, and a conical base is still a third

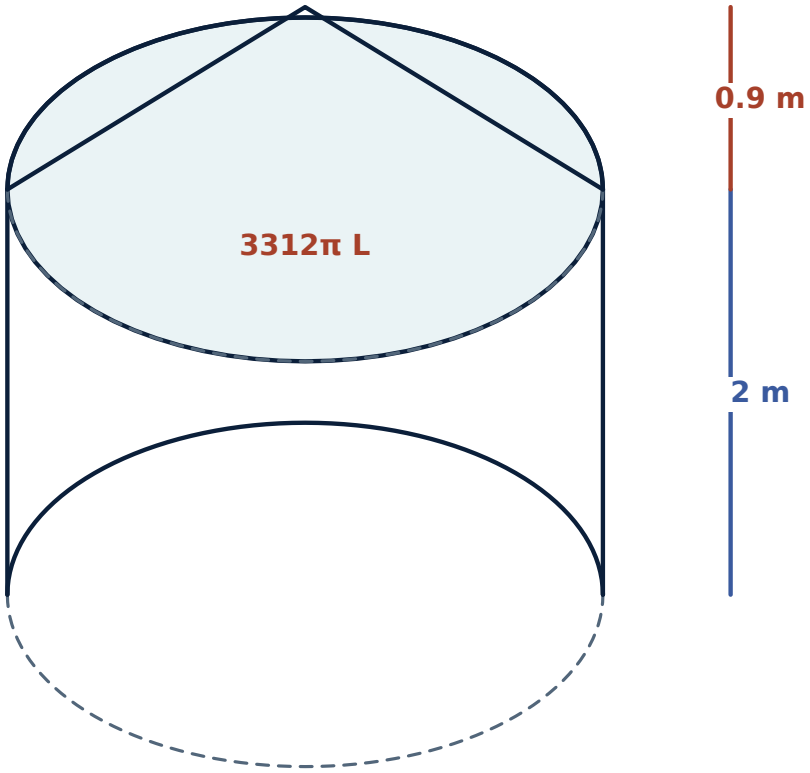
Real vessels are two shapes stuck together — a cylindrical body on a conical base, a tank with a conical top — and for capacity the rule could not be simpler: **the pieces add**. There is none of the care a compound solid's *surface* area needed in **G14**, because there is no hidden join to count: the inside of a vessel is all still inside. Split the vessel where one shape stops and the next begins, work each piece out, add, and cross the bridge once at the end. A cone holds a third of its own cylinder whichever way up it is drawn.

A vessel with a conical base, in cross-section



A cylindrical body 45 cm tall on a cone 30 cm deep, both of them 20 cm in radius — 40 cm across the top. Capacities **add***, with no thinking required: the body holds π × 400 × 45 = 18 000π cm³ and the cone below it holds ⅓ × π × 400 × 30 = 4 000π cm³, so the vessel holds 22 000π cm³ =* **22π L** *=* **69.1 L***. The cone is still a third of the cylinder it would fit inside, whichever way up it is drawn.*

A water tower: a cylinder with a conical top



Radius 1.2 m throughout: a cylinder 2 m tall with a cone 0.9 m tall on top of it. In cubic metres it holds 2.88π + 0.432π = 3.312π m³. Multiply by 1 000 once — not by 1 000 twice, and not by 1 000 000 — and the capacity is **3 312π L***, which is* **10 400 L** *to 3 significant figures. The radius is not printed on the drawing because a horizontal length on an isometric solid is not drawn to its true size; the two vertical dimension lines are.*

Worked example 5 · A vessel made of two pieces

1	A funnel has a cylindrical body 45 cm tall on a conical base 30 cm deep, both of radius 20 cm. **Calculate** the body's capacity.	**π × 400 × 45 = 18 000π cm³ = 18π L**
2	**Calculate** the cone's capacity.	**⅓ × π × 400 × 30 = 4 000π cm³ = 4π L**
3	**Calculate** the whole vessel's capacity, exactly and to 3 significant figures.	**22π L, which is 69.1 L**
4	Does it matter that the cone is upside down?	**no — a cone holds a third of its cylinder either way up**
5	A water tower is a cylinder of radius 1.2 m and height 2 m with a cone 0.9 m tall on top. **Calculate** its volume in cubic metres.	**2.88π + 0.432π = 3.312π m³**
6	**Calculate** its capacity in litres, to 3 significant figures.	**3 312π L, which is 10 400 L**

Capacities add, with none of the care that a compound solid's *surface* area needed in G14 — there is no hidden join to count, because the inside of a vessel is all still inside. Split the vessel where one shape stops and the next begins, name the pieces, work each one out and add. The one thing worth checking twice is the unit: the water tower is measured in metres, so it crosses the bridge by a thousand, once.

Vessel	The cylinder	The cone	Total	To 3 s.f.
Funnel: cylinder on a cone	18π L	4π L	22π L	69.1 L
Hopper: cylinder on a cone	6.3π L	1.575π L	7.875π L	24.7 L
Water tower: cylinder under a cone	2880π L	432π L	3312π L	10400 L
Small silo: cylinder under a cone	375π L	50π L	425π L	1340 L

EXAMPLE A silo is a cylinder of radius 0.5 m and height 1.5 m with a cone 0.6 m tall on top. The cylinder holds 0.375π m³ and the cone holds

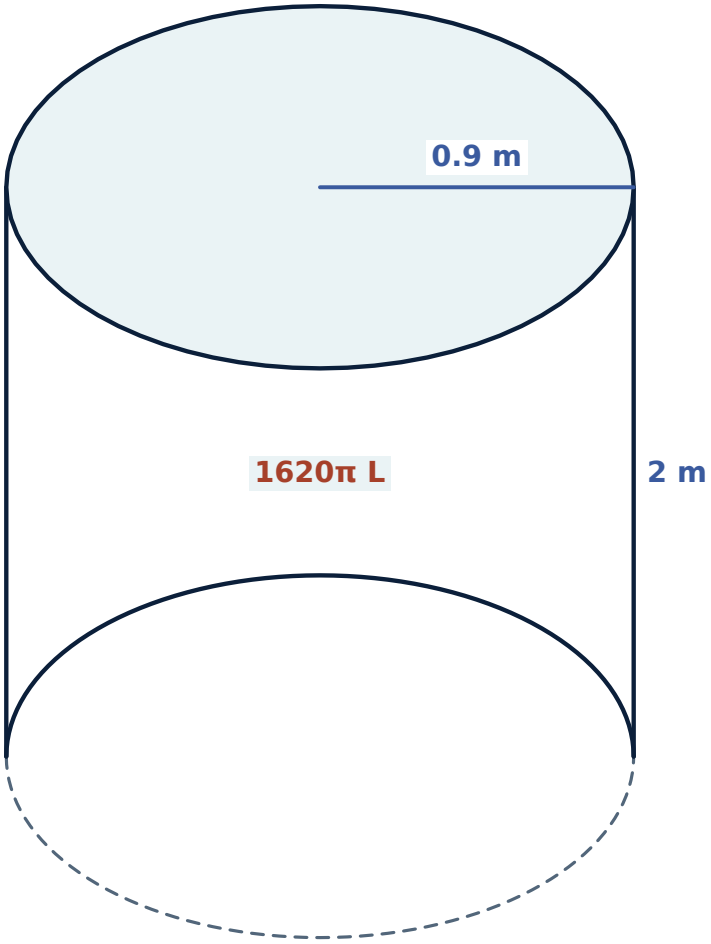
0.05π m³, so the silo holds 0.425π m³. Multiply by 1 000 once: **425π L**, which is **1 340 L** to 3 significant figures.

Concept 6 · Choosing the unit, and judging a stated capacity

A claim is checked against the dimensions and their bounds

Two judgements close the unit. The first is which **unit** to speak in: millilitres for a dose, litres for a bucket or a tank, cubic metres for a reservoir or a water meter — that is **N06**, and the test is whether you can say the number out loud. The second is harder and it is what *justify* means at Extended level. A capacity printed on the side of a tank is a **claim**, and you can test it: measure the tank, work the capacity, and then ask what the measurements could really have been. A claim that sits inside the window the tape allows is fair; one that sits outside it is not; and an answer quoted to more figures than the window permits is not defensible either.

The tank that says 5 000 litres on the side



Measured inside with a tape: 1.8 m across, so the radius is 0.9 m, and 2 m tall. That gives π × 0.81 × 2 = 1.62π m³, or **1 620π L — 5 090 L** *to 3 significant figures. The tank is not wrong and neither is the tape: the tape reads to the nearest 0.1 m, and once that is allowed for the true capacity lies anywhere between about 4 690 L and 5 510 L.* **5 000 L is inside that window**, *so the claim survives — and the only honest thing to say about the tank is that it holds about 5 000 litres.*

Worked example 6 · Justify the claim: does the tank really hold 5 000 litres?

1	A tank has “5 000 litres” printed on its side. Measured inside with a tape it is 1.8 m across and 2 m tall. **Write down** its radius.	**0.9 m**
2	**Calculate** its volume in cubic metres, exactly.	**π × 0.81 × 2 = 1.62π m³**
3	**Calculate** its capacity in litres, exactly and to 3 significant figures.	**1 620π L, which is 5 090 L**
4	The tape reads to the nearest 0.1 m. **Write down** the possible error in each measurement.	**0.05 m either way**
5	**Calculate** the smallest and the largest capacities the tank could have.	**4 690 L and 5 510 L**
6	**Justify** a verdict on the claim.	**5 000 L lies inside 4 690 L to 5 510 L, so the claim is fair**

This is what *justify* means at Extended level. The arithmetic alone says 5 090 L and invites you to announce that the tank is 90 litres over its label — but 5 090 came from two measurements each of which could be 0.05 m out, and once that is allowed for the honest answer is a window 800 litres wide. The claim sits comfortably inside it. A verdict is defensible when it survives the worst the measurements can do to it, and not before.

Stated	Measured	From the tape	Difference	The window the tape allows	Verdict
5000 L	1.8 m × 2 m	5090 L	−1.76%	4690 L to 5510 L	fair
2500 L	1.4 m × 1.6 m	2460 L	1.50%	2220 L to 2720 L	fair
210 L	0.56 m × 0.85 m	209 L	0.308%	204 L to 214 L	fair
1000 L	1 m × 1.2 m	942 L	6.10%	815 L to 1080 L	fair

Stated	Measured	From the tape	Difference	The window the tape allows	Verdict
5000 L	1.2 m × 1.5 m	1700 L	195%	1510 L to 1900 L	not fair

EXAMPLE A tank sold as a 1 000 litre tank measures 1 m across and 1.2 m tall inside. Its capacity is π × 0.25 × 1.2 = 0.3π m³ = **942 L**, which is 6.1% under the label. Before complaining, work the bounds: with the tape reading to the nearest 0.1 m the tank could hold anything from 815 L to 1 080 L, and 1 000 L sits inside that. The label is fair; the certainty was not.

The capacity routine

1. Find the volume first, in the unit the measurements came in — Do not convert anything yet. A tank measured in centimetres has a volume in cm^3; one measured in metres has a volume in m^3. Keep π exact. This step is G07 and G14, and nothing else.

2. Cross the bridge once — cm^3 to mL is not a conversion at all — the numbers are identical. mL to L is ÷ 1 000. m^3 to L is × 1 000. cm^3 to m^3 is ÷ 1 000 000, because it is two thousands at once. Write the step you are using down beside the arithmetic.

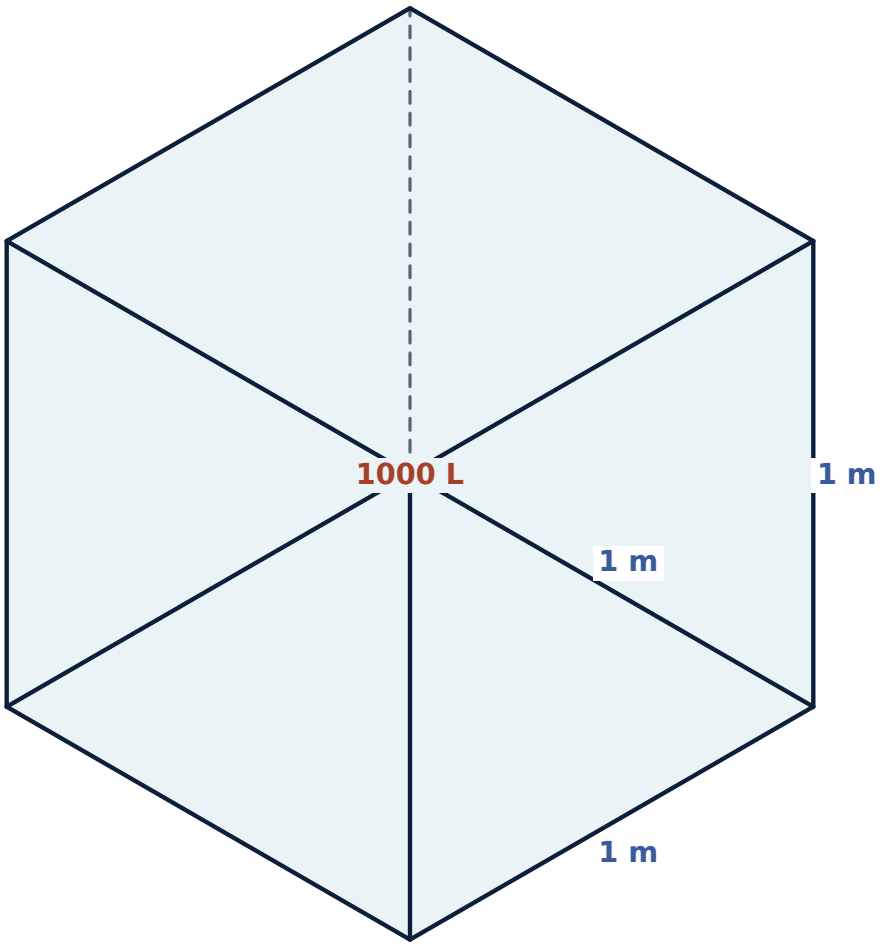
3. Ask whether the sides are straight — If the vessel is a prism or a cylinder, the fraction of the depth and the fraction of the capacity are the same number. If it is a cone or any vessel that narrows, they are not: the capacity fraction is the **cube** of the depth fraction.

✓ **Round last, and only as far as the measurements allow** — Give the exact answer in π first if you can, then a 3 significant figure decimal. Then look at the measurements: two figures on a tape cannot produce six figures of capacity, and a capacity claim that survives its own bounds is the only kind worth defending.

The mistakes that cost the most marks

Trap 1 · Turning a cubic metre into a thousand of anything

One cubic metre



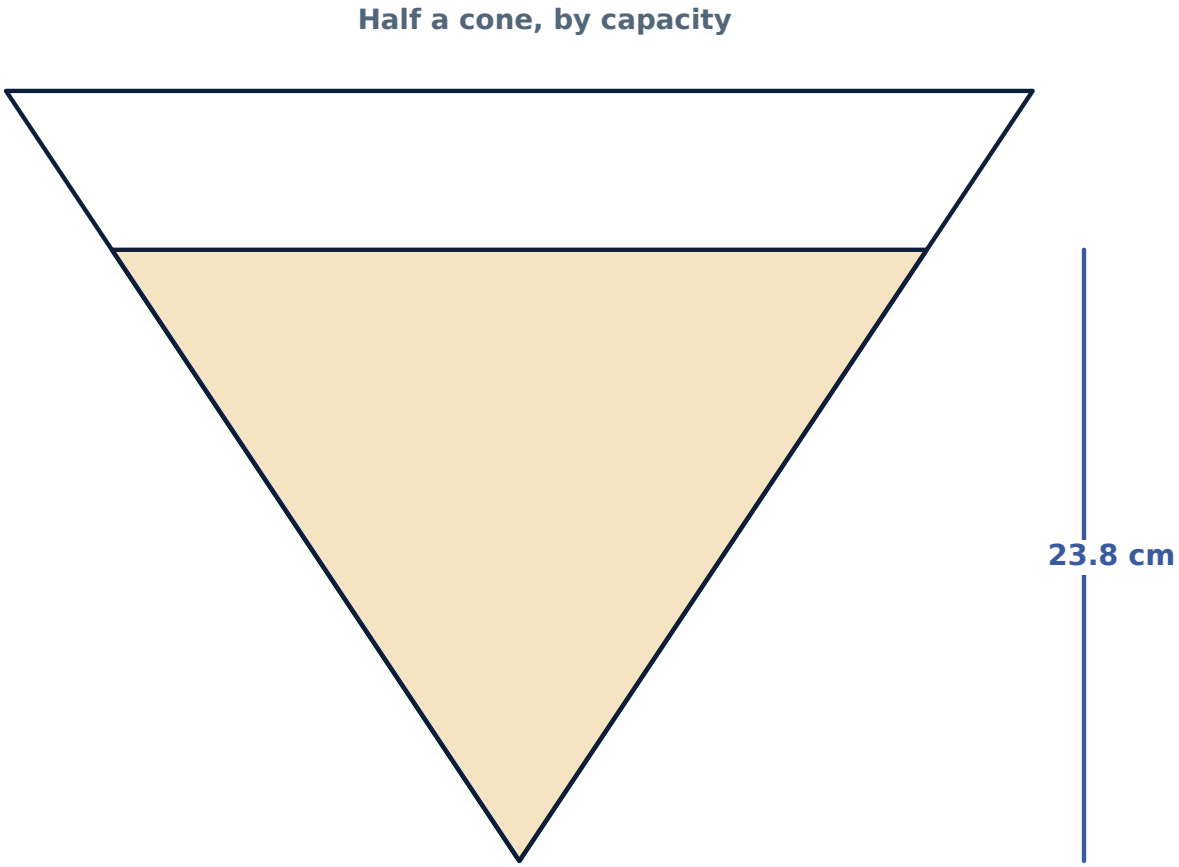
*A box one metre on every edge. Every edge is 100 cm, so it is 100 × 100 × 100 = 1 000 000 cm³, and at 1 cm³ to the millilitre that is 1 000 000 mL = **1 000 L**. A student who divides a cubic metre by 1 000 to reach cubic centimetres has shrunk this box to the size of a litre carton.*

WRONG “The reservoir holds 2.4 m³, and 1 m³ = 1 000 cm³, so that is 2 400 cm³, which is **2.4 L**.” A reservoir the size of a large bottle of cool drink.

RIGHT A metre is 100 cm, so a cubic metre is 100 × 100 × 100 = 1 000 000 cm³. That makes 2.4 m³ = 2 400 000 cm³ = 2 400 000 mL = **2 400 L**.

The wrong answer is a **thousand** times too small, which is the one size of error nobody can defend and everybody can spot. Two defences, and they cost nothing. First, do the cube out loud: a cubic metre is a cube of 100 cm, and 100³ is a million — the same reasoning that makes 1 m² = 10 000 cm² rather than 100, which is G11's whole subject. Second, sanity-check the size against something you have held: a cubic metre is a thousand litres, so it is roughly five full baths, and a 2.4 m³ reservoir is a serious tank and not a bottle. If your capacity would fit in a bag, and the vessel would not, go back to the power of ten.

Trap 2 · Assuming a cone half way up is a cone half full



This is the same 30 cm cone, filled with exactly half of what it can hold — and the water is **23.8 cm** *deep, not 15 cm. To halve the capacity you divide the depth by the cube root of 2, and the surface still sits nearly four fifths of the way up. Every drawing of a half-full cone that looks half-full is wrong.*

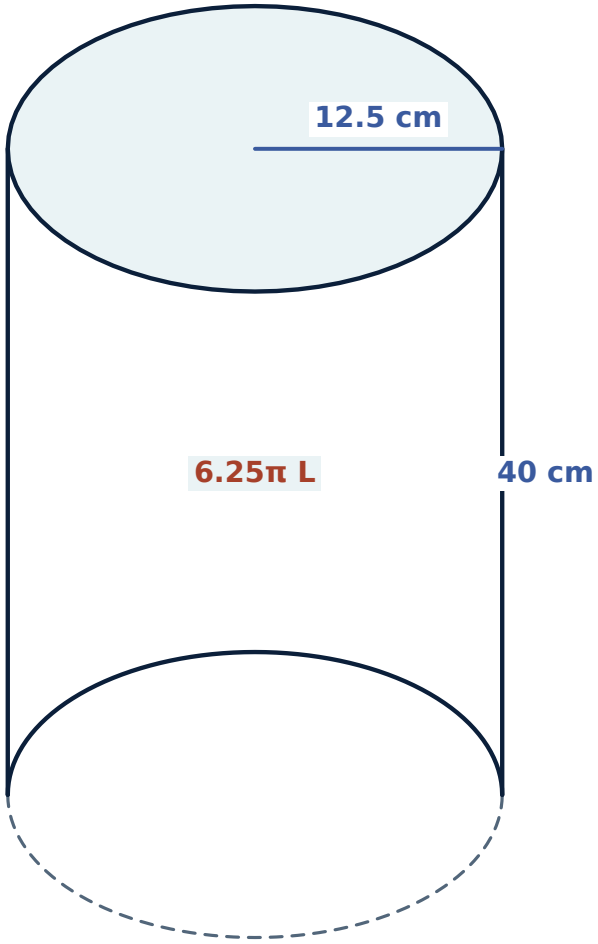
WRONG “The funnel holds 4π L and the water is 15 cm up a 30 cm cone, so it is half full — **2π L**, about 6.28 L.” The picture looks half full, and it is not.

RIGHT The water in a cone is a scaled copy of the whole cone. Half the depth means half the radius too, so the capacity is (½)³ = one eighth: π × 100 × 15 ÷ 3 = 500π cm³ = **0.5π L**, about 1.57 L.

The error here is a factor of four, and it always makes the answer too big. The defence is to stop looking at the picture and start writing down the surface radius: the moment you write “radius of the water = 10 cm” the calculation cannot go wrong, because the cone you are working out is the one that is actually there. It is worth knowing which way the surprise runs. To fill a cone to *half its capacity* the water has to reach 30 ÷ $\sqrt[3]{2}$ = **23.8 cm** — nearly four fifths of the way up. Every vessel that narrows towards the bottom does this, which is why measuring jugs have their marks crowded together at the bottom and spread out at the top.

Trap 3 · Quoting a capacity to more figures than the measurements can carry

The drum that was measured with a metre rule



25 cm across and 40 cm deep, each read to the nearest centimetre. The arithmetic gives 6.25π L exactly, and a calculator will happily print 19.634 954 08 litres. Two measurements of two figures each cannot support ten. To 3 significant figures it is **19.6 L***, and once the nearest-centimetre reading is allowed for the truth lies between 18.6 L and 20.7 L — so the honest sentence is about 20 litres.*

WRONG “The drum is 25 cm across and 40 cm deep, so it holds π × 12.5² × 40 = 19 634.954 08 cm³ = **19.634 954 08 L**.” Ten figures, from two measurements of two figures each.

RIGHT The exact capacity is 6.25π L, and to 3 significant figures that is **19.6 L**. Each measurement was read to the nearest centimetre, so the truth lies between 18.6 L and 20.7 L, and the honest sentence is *about 20 litres*.

A calculator will always offer more digits than a tape measure has earned, and copying them out is not accuracy — it is a claim about the measurements that the

measurements do not support. The discipline has two halves. Give the exact answer where there is one (6.25π L is exact and it is short), then a 3 significant figure decimal. Then, whenever a question asks you to **justify** a capacity, work the bounds: put the smallest possible measurements in, put the largest in, and see how many leading digits survive. Here the answer is **one**. That is the N18 *Trapped Between Bounds* argument, and Concept 6 does nothing else.

One step beyond

One step beyond — the argument this unit borrowed. Every verdict in Concept 6 rested on a pair of bounds, and this unit simply used them. Where they come from — why a length read to the nearest 0.1 m carries a possible error of 0.05 m, how bounds behave when quantities are multiplied, and how to decide how many figures of an answer survive — is **N18 Trapped Between Bounds** in the Number strand's Extended band. If Strand iv of the Criterion D task felt like the hardest part of this unit, that is the page to read. Two more links are worth knowing about. The cube law that makes a cone at half its depth only an eighth full is **G11 Convert It** and **G18**, and it is the same k^3 that scaled the tanks in P20. And the volumes themselves — $\pi r^2 h$ and a third of it — are **G07** and **G14**, which this unit uses on every page and reteaches nowhere.

Practice · Fluency · 24 questions

P1	**Convert** 4 200 cm³ to litres.
P2	**Convert** 0.65 L to millilitres.
P3	**Convert** 3.5 m³ to litres.
P4	**Convert** 1 750 mL to litres.
P11	A cylindrical bucket has radius 15 cm and height 28 cm. **Calculate** its capacity in litres, exactly and to 3 significant figures.
P12	A cylindrical flask has radius 9 cm and height 50 cm. **Calculate** its capacity in litres, exactly and to 3 significant figures.
P13	A storage box measures 24 cm by 18 cm by 15 cm inside. **Calculate** its capacity in litres.
P14	A conical funnel has radius 12 cm and depth 21 cm. **Calculate** its capacity in litres, exactly and to 3 significant figures.
P21	A drum of radius 20 cm and height 60 cm holds water 45 cm deep. **Calculate** the capacity it is holding, exactly and to 3 significant figures.
P22	**Calculate** the percentage of that same drum that is full.
P23	A conical vessel is 24 cm deep and 9 cm in radius at the rim. It is filled to a depth of 12 cm. **Calculate** the capacity it is holding, exactly and to 3 significant figures.
P24	**Calculate** the percentage of that same conical vessel that is full.
P31	A tank holds 240 L and a tap fills it at 8 litres a minute. **Calculate** the time it takes to fill.
P32	A tap runs at 12 litres a minute for 25 minutes. **Calculate** the capacity it has delivered.
P33	A pump delivers 1 500 L in 50 minutes. **Calculate** its rate in litres a minute.
P34	Water runs into a tank at 20 litres a minute and leaks out at 5 litres a minute. **Write down** the net rate.

P41 A vessel is a cylinder of radius 15 cm and height 28 cm standing on a cone of radius 15 cm and depth 21 cm. **Calculate** its capacity, exactly and to 3 significant figures.

P42 A silo is a cylinder of radius 0.5 m and height 1.5 m with a cone 0.6 m tall on top. **Calculate** its capacity in litres, exactly and to 3 significant figures.

P43 A vessel is a cylinder of radius 6 cm and height 10 cm on a cone of radius 6 cm and depth 10 cm. **Calculate** its capacity, exactly and to 3 significant figures.

P44 A funnel is a cylinder of radius 12 cm and height 35 cm on a cone of radius 12 cm and depth 21 cm. **Calculate** its capacity, exactly and to 3 significant figures.

P51 **State** the sensible unit for the dose in a medicine spoon, and give the size you would expect.

P52 **State** the sensible unit for the capacity of a household water tank, and give the size you would expect.

P53 A tank is 1 m across and 1.2 m tall inside. **Calculate** its capacity in litres, exactly and to 3 significant figures.

P54 That tank has “1 000 litres” printed on it. **Calculate** the percentage by which the claim exceeds the capacity the measurements give, to 3 significant figures.

Practice · Applied · 24 questions

P5 A bottle holds 500 mL. **Determine** how many full bottles it takes to fill a 12 L container.

P6 A rectangular trough measures 120 cm by 45 cm by 30 cm inside. **Calculate** its capacity in litres.

P7 A tanker carries 18 m³. **Convert** that to litres and to cubic centimetres.

P8 A medicine spoon holds 5 mL. **Write down** that capacity in cubic centimetres and in litres.

P15 A cylindrical tank is 1.6 m across and 1.5 m tall inside. **Calculate** its capacity in litres, exactly and to 3 significant figures.

P16 A cylindrical bucket has radius 12 cm and height 35 cm. **Calculate** its capacity, and **determine** how many full bucketfuls a 200 L drum takes.

P17 A conical hopper has radius 15 cm and depth 21 cm. **Calculate** its capacity in litres, exactly and to 3 significant figures.

P18 A tank with a rectangular base 150 cm by 60 cm holds 450 L. **Calculate** its depth.

P25 A drum of radius 25 cm and height 90 cm holds water 63 cm deep. **Calculate** the capacity it is holding and the capacity still empty.

P26 A drum of radius 30 cm is holding 100 L of water. **Calculate** the depth of the water, to 3 significant figures.

P27 A conical vessel is 15 cm deep and 10 cm in radius at the rim, and is filled to a depth of 9 cm. **Calculate** the capacity held and the percentage full.

P28 A conical vessel is 30 cm deep. **Calculate** the depth to which it must be filled to hold exactly a quarter of its capacity, to 3 significant figures.

P35 That same tank holds 600 L and starts empty. **Calculate** how long it takes to fill while it is leaking.

P36 A tank of radius 30 cm and height 80 cm is filled at 6 litres a minute. **Calculate** the time it takes, exactly and to 3 significant figures.

P37 A drum of radius 20 cm empties so that the water level falls a steady 2 cm a minute. **Calculate** the flow in litres a minute, exactly and to 3 significant figures.

P38 A drum 90 cm deep drains at a steady 1.5 cm a minute. **Calculate** how long it takes to empty, and the depth left after 20 minutes.

P45 The funnel of Worked example 5 — a cylinder 45 cm tall on a cone 30 cm deep, both of radius 20 cm — is filled exactly to the top of its cone. **Calculate** the capacity it is holding.

P46 The same funnel is filled to a depth of 15 cm. **Calculate** the capacity it is holding, and **determine** what percentage of the whole funnel that is.

P47 A water tower is a cylinder of radius 1.2 m and height 2 m with a cone 0.9 m tall on top. **Calculate** its capacity in litres, and **determine** how many full 210 L drums it would fill.

P48 The hopper of P41 is emptied at 3 litres a minute. **Calculate** how long it takes, to 3 significant figures.

P55 A rainwater tank measures 1.4 m across and 1.6 m tall inside and is sold as a 2 500 litre tank. **Calculate** the capacity its measurements give, exactly and to 3 significant figures.

P56 The tape used on that tank reads to the nearest 0.1 m. **Calculate** the smallest capacity the tank could have, to 3 significant figures.

P57 A different tank is sold as a 5 000 litre tank and measures 1.2 m across and 1.5 m tall inside. **Calculate** the capacity its measurements give, exactly and to 3 significant figures.

P58 A drum measures 0.56 m across and 0.85 m tall inside and is called a 210 litre drum. **Calculate** the percentage by which the name exceeds the measured capacity, to 3 significant figures.

Practice · Challenge · 12 questions

P9 A cube-shaped container holds exactly 8 L. **Calculate** the length of one of its edges.

P10 A box holds exactly 1 m^3. **Determine** how many 10 cm cubes fit inside it, and **justify** your answer by converting.

P19 A cylindrical tank is to hold 2 000 L and to stand 2 m tall. **Calculate** the radius it needs, in metres, to 3 significant figures.

P20 Two tanks are the same shape, and every length of the larger is 1.5 times the matching length of the smaller. The smaller holds 240 L. **Calculate** the capacity of the larger, and **justify** the factor you used.

P29 A conical vessel is 30 cm deep and 20 cm in radius at the rim, and is filled to a depth of 6 cm. **Calculate** the capacity held and the percentage full, and **explain** why the percentage is so small.

P30 A drum and a conical vessel are both 60 cm deep and both hold 30 L when full. Both are filled to a depth of 30 cm. **Calculate** the difference between the capacities they are holding.

P39 A 1 200 L reservoir fills at 25 litres a minute while leaking at 4 litres a minute. After 20 minutes the leak is repaired. **Calculate** the total time to fill.

P40 A conical vessel 24 cm deep and 9 cm in radius at the rim is filled at 0.05 litres a minute. **Calculate** how long it takes to fill completely, to 3 significant figures.

P49 The funnel of Worked example 5 is filled at 4 litres a minute. **Calculate** the time to fill the cone alone and the time to fill the whole vessel, exactly and to 3 significant figures.

P50 A vessel is a cylinder of radius 10 cm and height 15 cm on a cone of radius 10 cm and depth 15 cm. **Show that** its capacity is exactly four thirds of the cylinder's, and **justify** why that is true for any such vessel.

P59 A tank measured as 1.8 m across and 2 m tall, to the nearest 0.1 m, gives a capacity of 5 090 L. **Determine** how many significant figures of that answer can be defended, and **justify** your decision.

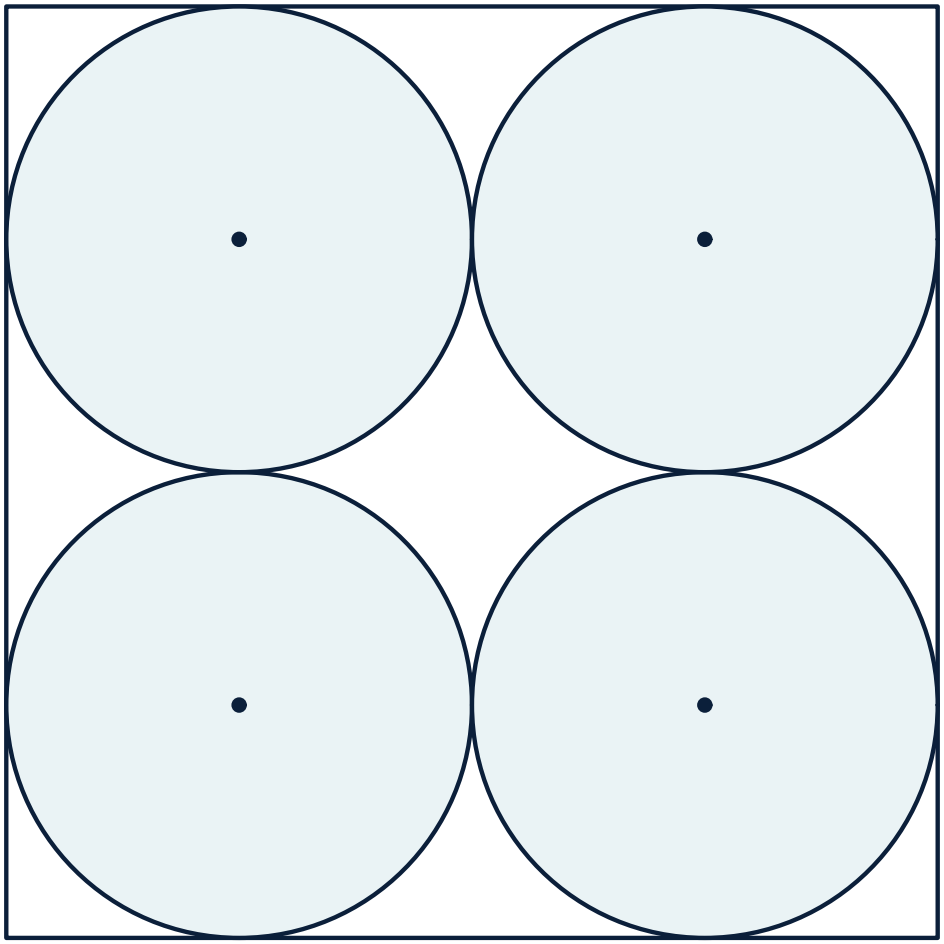
P60 A reservoir holds 2.4 m^3 and is to be emptied into 210 litre drums. **Determine** how many drums are needed, and **justify** the rounding you chose.

Criterion B · Investigating patterns

Wasted space

A shop packs identical cylindrical tins into a cube-shaped crate 48 cm on every edge. Each tin is exactly as tall as it is wide, and the tins are packed so that they touch each other and touch the walls. Some of the crate is tin and the rest is air, and the air is **wasted capacity** — it is paid for, carried and stored, and it holds nothing. So the packer asks the obvious question: **if we used smaller tins, would we waste less?** Almost everybody says yes. You can test it in an afternoon with a cardboard box and some tins, or with rice and a measuring jug, and what you find is worth the trouble.

A crate of tins, seen from above



Looking down into the crate: four tins in this layer, and two layers below this one.

The crate is a cube 48 cm on every edge. These tins are 12 cm in radius and 24 cm tall, so two fit across, two fit front to back and two fit one on top of the other — eight tins in all, and they touch. The question of the investigation is how much of the crate the air

between them wastes, and what happens to that waste when the tins are made smaller.

Part 1 · structured

n	Tins	One tin	All the tins	Percentage wasted
1	1	27.648π L	27.648π L	21.5%
2				
4				
8				

The crate is a cube 48 cm on every edge and n is the number of tins along each edge. Each tin is exactly as tall as it is wide, so its radius is 48 ÷ 2n and there are n^3 of them. The n = 1 row is filled in for you, and the n = 8 row is the one you are asked to predict before you calculate anything.

1. **Predict**, before you calculate anything at all, what fraction of the crate is wasted when 8 tins are packed along every edge instead of 1. Write the prediction down where you will not be tempted to change it, and write down the reason you chose it.

2. Complete the table for n = 1, 2 and 4, where n is the number of tins along each edge of the crate. Give each tin's capacity in litres, the capacity of all the tins together, and the fraction of the crate that is wasted. If you can, build one case out of real tins and check it by filling the gaps with rice.

3. **Identify the pattern** in your table. Say what happens to one tin's capacity as n doubles, what happens to the number of tins, and what happens to the two of them multiplied together.

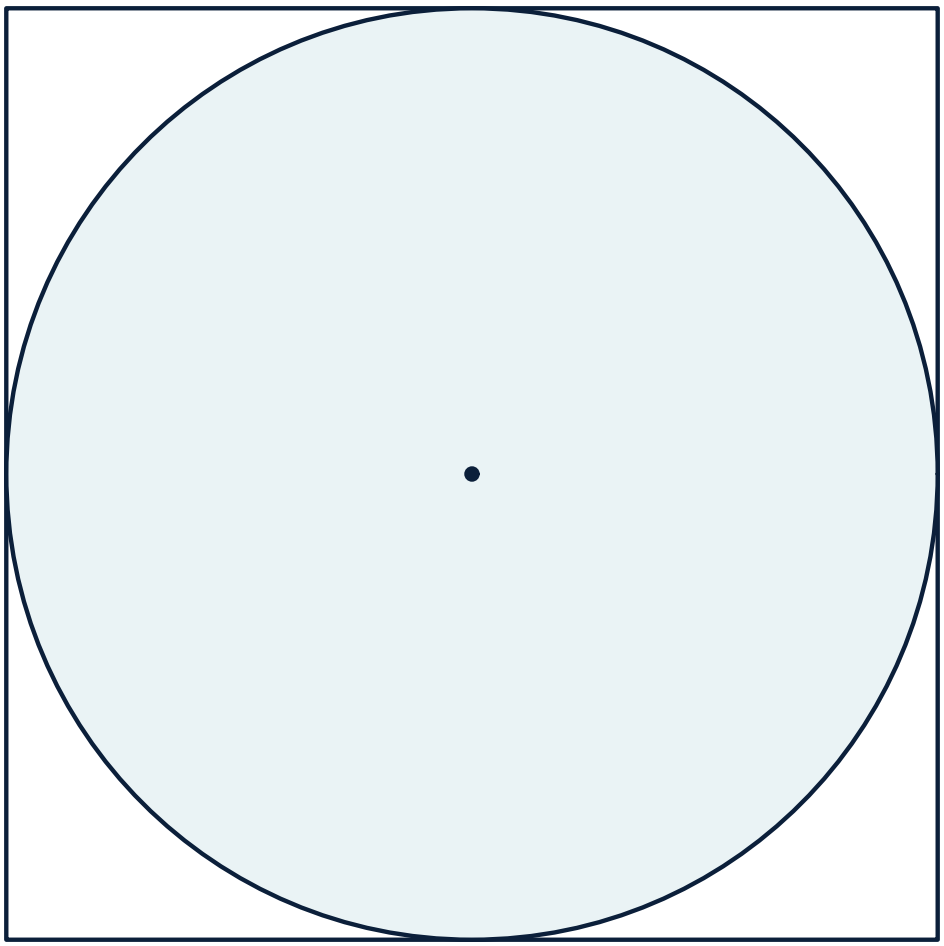
4. **Determine a general rule** for the fraction of the crate that is wasted, in terms of n. Then say what your rule gives as n grows very large, and whether n appears in it at all.

5. **Test and verify** your rule at n = 8: work the capacities out from scratch, without using the rule, and check that they agree with it. Compare the answer with the prediction you wrote in step 1 and say what went wrong with it.

6. **Justify** the rule. Every tin sits alone in a cubic cell exactly as wide as the tin, and the crate is nothing but a stack of identical cells. Work out what fraction of one cell the tin fills, with the radius left as a letter rather than a number, and explain why that settles the whole question — and

hence why the wasted fraction settles at a fixed value rather than shrinking towards zero.

One tin in its own square cell



24 cm

Every tin, whatever its size, sits alone in a square cell exactly as wide as the tin. That is the picture the whole investigation turns on: if you can say what fraction of one cell the tin fills, you have said what fraction of the whole crate the tins fill — because the crate is nothing but a stack of identical cells.

Part 2 · open

Now you choose what to investigate. Take one of these openings or propose your own. You decide how many cases you need, how to record what you find and how you will test it.

1. **Change the packing.** Stagger alternate rows so each tin sits in the hollow between two tins below it, instead of stacking them squarely. Investigate what that does to the wasted fraction: tabulate, predict a case you have not tried, determine a rule and **justify** why the answer is or is not independent of the tin size.

2. **Change the shape of the tin.** Keep the crate the same and make the tins twice as tall as they are wide, then three times, then half. Tabulate the wasted fraction, predict the next case, determine the rule in terms of the height-to-width ratio and **justify** what you find.

3. **Change the container.** Pack spheres instead of tins, in a cubic arrangement. Tabulate the wasted fraction for several sizes, determine the rule, compare it with the tin answer and **justify** the difference between the two numbers.

Whichever you choose, take it through the same ladder: **predict**, **identify** the pattern, **determine** a general rule, **test and verify** it on a case you have not already tabulated, and **justify** why it must be true.

Marked out of 8 against Criterion B. Completing the table earns the lower bands; the general rule, the test on a case outside the table and the one-cell argument with the radius left as a letter are what reach levels 5–8. Part 2 must show your own choice of method.

Remember, you should communicate in an organized and coherent manner.

Criterion D · Applying maths in real-life contexts

The meter, the tank and the drums

A household in a Gaborone yard reads its own water meter on the first and the last day of a month. The meter counts in **cubic metres**, the tank in the corner is sold in **litres**, and the drums beside it are called by a number nobody has ever checked — so before anything can be said about how much water the household uses, three different units have to be brought onto the same ladder. Every figure on the sheet below is one you could take yourself: two meter readings, a tape measure round the inside of a tank, a rain gauge and a count of the people in the yard. The sheet carries far more of it than the calculations need. Your job is to work out what the household actually uses in a day, and then to **justify** every part of that figure — the conversion, the precision, and whether the number is believable for the household described.

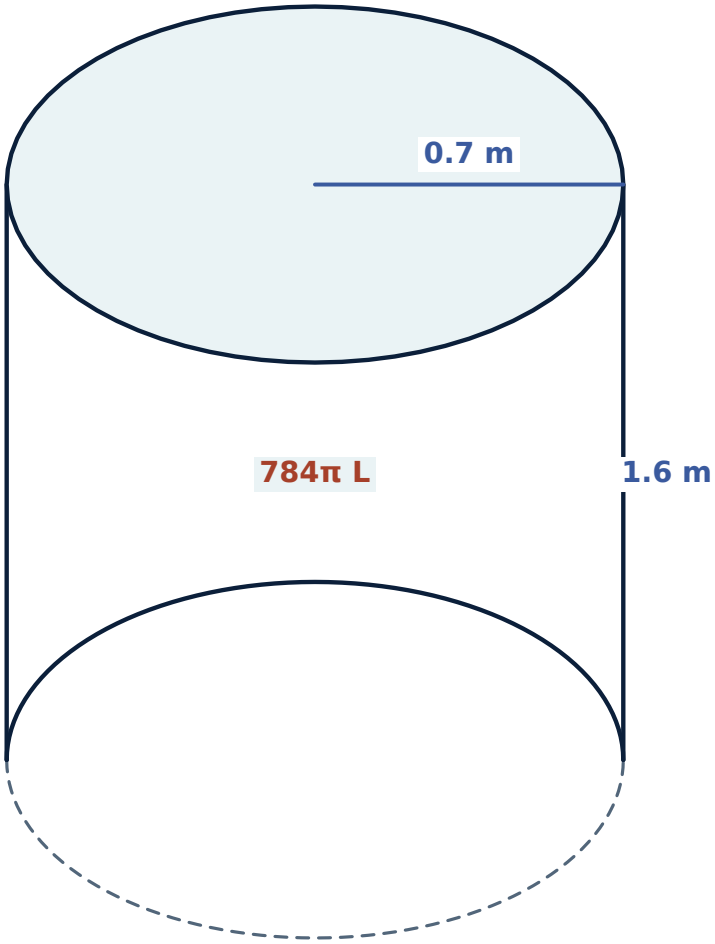
The measurement sheet (illustrative figures, written for this task)

- **Water meter reading on the 1st of the month:** 148 m³
- **Water meter reading on the 31st:** 161 m³
- **What the meter counts in:** whole cubic metres
- **Days between the two readings:** 30
- **People living in the yard:** 6
- **Rainwater tank, measured inside with a tape:** 1.4 m across, 1.6 m tall
- **What the label on the tank says:** 2 500 litres
- **Drums standing beside the tank:** 3
- **One drum, measured inside:** 0.56 m across, 0.85 m tall
- **What people call a drum of that shape:** a 210-litre drum
- **Roof area that drains into the tank:** 45 m²
- **Rain caught in the yard gauge during one storm:** 20 mm
- **Taps in the yard:** 4
- **Height of the stand the tank sits on:** 1.1 m

- **Buckets carried to the vegetable beds on a Saturday:** 9
- **Size of one bucket:** 20 litres
- **Vegetable beds in the yard:** 5

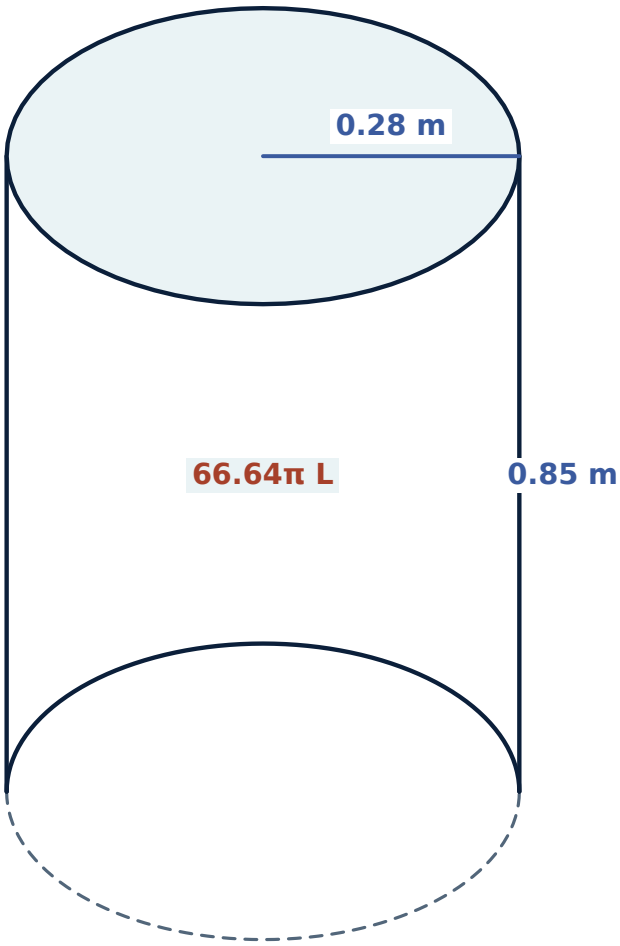
These figures are illustrative. The yard, the meter readings, the tank and the measurement sheet are an invented illustrative example written for the task. It is not taken from any business, price list or publication, and no organisation, person or source is named or implied. What is authentic is the shape of the problem: a water meter really does count in cubic metres, a tank really is sold by the litre, and a drum really is called by a number nobody has checked.

The rainwater tank in the yard



Measured inside with a tape: 1.4 m across and 1.6 m tall, so the radius is 0.7 m. That gives π × 0.49 × 1.6 = 0.784π m³, or **784π L** *— 2 460 L to 3 significant figures. The label on the tank says 2 500 litres, and Strand v is about whether that is a fair thing for it to say.*

One of the drums beside it



0.56 m across and 0.85 m tall inside, so the radius is 0.28 m and the capacity is 66.64π L, which is **209 L** *to 3 significant figures. Drums of this shape are usually called 210-litre drums, and 209 is what the tape actually gives — a difference far smaller than the accuracy of the tape itself.*

Identify which figures on the sheet are needed for the household's daily water use and which are needed for the capacity of the tank, and **justify** why each of the others cannot change either answer. *(Strand i — identifying the elements of the authentic situation)*

Select the strategies you will use and **state** them before you calculate: which subtraction, which conversion, which division, and which formula for the tank. For each conversion say which power of ten it needs and **justify** that choice. *(Strand ii — selecting adequate mathematical strategies)*

Apply them: find the water used in the month in cubic metres and in litres, the average use per day, the average use per person per day, the capacity of the tank exactly and to 3 significant figures, the capacity of one drum, and how many tankfuls the month's use amounts to. *(Strand iii — applying the strategies to reach a solution)*

Justify the degree of accuracy of your answers. The meter counts in whole cubic metres, so each reading carries a possible error of 0.5 m^3 and the difference between them carries a whole cubic metre. The tape reads to the nearest 0.1 m. Work out the smallest and the largest daily use the readings allow, and the smallest and largest capacity the tank could have, and **justify** how many figures of each answer are worth writing down. *(Strand iv — explaining the degree of accuracy of the solution)*

Justify whether the answers make sense for this household. Is the daily figure believable for six people, and what would it be for each person? Is the label on the tank a fair thing to print, given the bounds you found? How much water would the roof send into the tank in the storm on the sheet, and would that make a serious difference to a month's use? And **justify** what you would say to somebody who read the meter's 13 cubic metres as 13 litres. *(Strand v — explaining whether the solution makes sense in context)*

Submit: your list of which figures matter and which do not, the month’s use in cubic metres and in litres, the daily use and the daily use per person, the capacity of the tank exactly and to 3 significant figures, the capacity of one drum, the smallest and largest daily use the meter allows, and written answers for strands i, iv and v. **Marked out of 8** against Criterion D at **Extended** level, where the command term is **justify**. State every capacity in litres and every volume in m^3.

Answers

How the P-numbers run. The 60 practice items are numbered P1 to P60 in the order they appear in the lesson — ten per concept, and inside each concept four Fluency, then four Applied, then two Challenge. The same P-number identifies the item in the lesson, in this key and in the teacher pack, so the numbers inside each tier below step in blocks of ten rather than running consecutively: Fluency is P1–P4, P11–P14, P21–P24 and so on; Applied is P5–P8, P15–P18, …; Challenge is P9–P10, P19–P20, … Every answer carries its unit, and every answer that can be exact is given exactly first and rounded second.

Fluency · F-tier (24)

P1	**4.2 L**
P2	**650 mL**
P3	**3 500 L**
P4	**1.75 L**
P11	**6.3π L, which is 19.8 L**
P12	**4.05π L, which is 12.7 L**
P13	**6.48 L**
P14	**1.008π L, which is 3.17 L**
P21	**18π L, which is 56.5 L**
P22	**75%**
P23	**0.081π L, which is 0.254 L**
P24	**12.5%**
P31	**30 min**
P32	**300 L**
P33	**30 L/min**
P34	**15 L/min**
P41	**7.875π L, which is 24.7 L**

P42	**425π L, which is 1 340 L**
P43	**0.48π L, which is 1.51 L**
P44	**6.048π L, which is 19.0 L**
P51	**millilitres — about 5 mL**
P52	**litres — a few thousand L**
P53	**300π L, which is 942 L**
P54	**6.10%**

Applied — fully worked (24)

P5 24 bottles, since 24 × 0.5 L = 12 L

Bring both capacities onto the same rung of the ladder before dividing: 500 mL is 0.5 L, or 12 L is 12 000 mL — either works, provided you do not mix them. Then 12 ÷ 0.5 = 24, and because 24 is a whole number the last bottle fills the container exactly. Dividing 12 by 500 without converting gives 0.024, which is the answer to a question nobody asked.

P6 162 L

Volume first, in the unit the measurements came in: 120 × 45 × 30 = 162 000 cm³. Then cross the bridge once, dividing by 1 000, which gives 162 L. Check the size of the answer against something you have seen: a trough a bit over a metre long holding about 160 litres is believable, and 162 000 L would not be.

P7 18 000 L, and 18 000 000 cm³

Two different steps from the same starting point. Cubic metres to litres is × 1 000, so 18 m³ = 18 000 L. Cubic metres to cubic centimetres is × 1 000 000, because a metre is 100 cm and $100^3 = 1\,000\,000$, so 18 m³ = 18 000 000 cm³. The second answer is exactly a thousand times the first, which is what the ladder says it must be.

P8 5 cm³, and 0.005 L

The first step is not a conversion at all: 5 mL is 5 cm³, because a millilitre and a cubic centimetre are the same thing. The second is a division by a thousand, giving 0.005 L. Notice how badly litres suit this quantity — 0.005 L is a number nobody would say out loud, which is exactly why doses are written in millilitres.

P15 960π L, which is 3 020 L

The measurement given is the **diameter**, so the radius is 1.6 ÷ 2 = 0.8 m. The volume is π × 0.8² × 1.5 = 0.96π m³, and the tank is measured in metres, so the bridge is × 1 000: 960π L. To 3 significant figures that is 3 020 L. Reading the diameter as a radius would have given four times as much.

P16 5.04π L, which is 15.8 L; so 12 full bucketfuls

The bucket holds π × 12² × 35 = 5 040π cm³, and crossing the bridge once gives 5.04π L = 15.8 L. Then 200 ÷ 15.8 = 12.6, and a part bucketful does not count, so the answer rounds **down** to 12 full bucketfuls with about 10 L still to go. Rounding a count is a decision about the situation, not about the arithmetic.

P17 1.575π L, which is 4.95 L

A cone is a third of its cylinder, so the volume is ⅓ × π × 15² × 21 = 1 575π cm³. Cross the bridge once: 1.575π L, which is 4.95 L to 3 significant figures. The cylinder of the same radius and height would hold three times as much, 4.725π L, which is a useful check on the size of the answer.

P18 50 cm, which is 0.5 m

Work in one unit throughout. 450 L is 450 × 1 000 = 450 000 cm³, and the base area is 150 × 60 = 9 000 cm², so the depth is 450 000 ÷ 9 000 = 50 cm. In metres the same sum reads 0.45 m³ ÷ (1.5 × 0.6) m² = 0.5 m, which is the same answer reached from the other side of the bridge.

P25 39.375π L, which is 124 L; and 16.875π L, which is 53.0 L, still empty

Straight sides, so the water is simply a shorter drum: π × 625 × 63 = 39 375π cm³ = 39.375π L = 124 L. The whole drum holds 56.25π L, so what is left is 56.25π − 39.375π = 16.875π L = 53.0 L. As a check, 63 ÷ 90 = 70%, and 30% of 56.25π is indeed 16.875π.

P26 35.4 cm

Cross the bridge first, in the direction that gives cubic centimetres: 100 L = 100 000 cm³. The base area is π × 30² = 900π cm², so the depth is 100 000 ÷ 900π = 1 000 ÷ 9π = 35.4 cm to 3 significant figures. Keep the π in the working and round once at the end.

P27 0.108π L, which is 0.339 L; and 21.6% full

First the surface radius: the water is a scaled cone, so its radius is 10 × 9 ÷ 15 = 6 cm. Then the capacity is ⅓ × π × 6² × 9 = 108π cm³ = 0.108π L = 0.339 L. The percentage is the cube of the depth fraction: (9 ÷ 15)³ =

$0.6^3 = 0.216$, so 21.6% — not the 60% the depth alone suggests.

P28 18.9 cm

Going backwards through the cube law means a cube root. The capacity fraction is the cube of the depth fraction, so the depth fraction is the cube root of the capacity fraction: $30 \times \sqrt[3]{(1 \div 4)} = 30 \div \sqrt[3]{4} = 18.9$ cm to 3 significant figures. Notice that a quarter of the capacity still reaches nearly two thirds of the way up.

P35 40 min

The leak does not need a separate calculation — it changes the rate, not the capacity. The net rate is $20 - 5 = 15$ L/min, so the time is $600 \div 15 = 40$ min. Without the leak it would have taken $600 \div 20 = 30$ min, so the leak costs ten minutes and wastes $5 \times 40 = 200$ L on the way.

P36 12π min, which is 37.7 min

First the capacity: $\pi \times 900 \times 80 = 72\,000\pi$ cm³ $= 72\pi$ L. Then the time is $72\pi \div 6 = 12\pi$ min, which is 37.7 min to 3 significant figures. Keeping the π makes the division trivial and the rounding happens once, at the end, exactly as the house rule asks.

P37 0.8π L/min, which is 2.51 L/min

Every minute the drum loses a slice of water 2 cm thick, and that slice is itself a cylinder: $\pi \times 20^2 \times 2 = 800\pi$ cm³. Cross the bridge once and the flow is 0.8π L/min, which is 2.51 L/min. Because the sides are straight, the slice is the same every minute, which is why a steady fall in depth means a steady flow.

P38 60 min; and 60 cm left

The depth alone answers both parts, because the rate is given as a rate of depth: $90 \div 1.5 = 60$ min to empty, and after 20 minutes the level has fallen $1.5 \times 20 = 30$ cm, leaving $90 - 30 = 60$ cm. Nothing here needs the radius at all — a good reminder to read what the rate is a rate *of*.

P45 4π L, which is 12.6 L

Filled to the top of the cone means the cone is full and the cylinder above it is empty, so only one piece counts: $\frac{1}{3} \times \pi \times 400 \times 30 = 4\,000\pi$ cm³ $= 4\pi$ L $= 12.6$ L. The whole funnel holds 22π L, so the cone that fills the bottom two fifths of the height holds under a fifth of the capacity.

P46 0.5π L, which is 1.57 L; and 2.27% of the funnel

At 15 cm the water is entirely inside the cone, and it is a scaled cone of radius 10 cm: $\frac{1}{3} \times \pi \times 100 \times 15 = 500\pi$ cm³ $= 0.5\pi$ L $= 1.57$ L. As a fraction of the whole funnel that is $0.5\pi \div 22\pi = 1 \div 44 = 2.27\%$. The π

cancels, which is a good sign that the fraction is exact.

P47 3 312π L, which is 10 400 L; and 49 full drums

The two pieces are 2.88π m³ and 0.432π m³, so the vessel holds 3.312π m³. The measurements are in metres, so the bridge is × 1 000 exactly once: 3 312π L = 10 400 L to 3 significant figures. Then 10 405 ÷ 210 = 49.5, and half a drum is not a full drum, so it rounds down to 49.

P48 8.25 min

The capacity was 7.875π L = 24.7 L, so the time is 7.875π ÷ 3 = 2.625π min = 8.25 min to 3 significant figures. It is worth doing the division on the exact form: the π simply comes along, and the single rounding at the end keeps all three significant figures honest.

P55 784π L, which is 2 460 L

The radius is 1.4 ÷ 2 = 0.7 m, so the volume is π × 0.49 × 1.6 = 0.784π m³. In metres the bridge is × 1 000, so the capacity is 784π L = 2 460 L to 3 significant figures. That is about 1.5% under the 2 500 printed on it — but P56 shows why that is not yet a reason to complain.

P56 2 220 L

A reading to the nearest 0.1 m carries a possible error of 0.05 m either way, so the smallest tank consistent with the readings is 1.35 m across and 1.55 m tall. That gives π × 0.675² × 1.55 = 2 220 L to 3 significant figures. The largest is 1.45 m by 1.65 m, which gives 2 720 L, so the 2 500 L claim sits comfortably inside the window. This is **N18**'s argument, used and not reproved.

P57 540π L, which is 1 700 L

The radius is 0.6 m, so the volume is π × 0.36 × 1.5 = 0.54π m³, and the capacity is 540π L = 1 700 L to 3 significant figures. Even the largest tank the tape allows — 1.25 m by 1.55 m — holds only 1 900 L, so 5 000 L is nowhere near the window and this claim cannot be defended at all.

P58 0.308%

The radius is 0.28 m, so the capacity is π × 0.0784 × 0.85 × 1 000 = 66.64π L = 209 L to 3 significant figures. The name is 210 L, so the difference is (210 − 209.36) ÷ 209.36 = 0.308%. Three tenths of one per cent is far smaller than anything a tape measure could resolve, so the name is a fair one.

Challenge — fully worked (12)

P9 20 cm

Cross the bridge downwards first: 8 L = 8 000 cm³. The container is a cube, so its edge is the cube root of its volume, $\sqrt[3]{8\,000} = 20$ cm. Check it forwards: 20 × 20 × 20 = 8 000 cm³ = 8 L. Notice that eight litre cubes stacked two by two by two make exactly this container, and its edge is *twice* the litre cube's 10 cm, not eight times it — the cube law again.

P10 1 000 of them, because 1 m³ = 1 000 L and each 10 cm cube holds 1 L

Count them first: the box is 100 cm on every edge, so 10 fit along each edge and 10 × 10 × 10 = 1 000 fit inside. Now convert instead: 1 m³ = 1 000 L, and each 10 cm cube holds exactly 1 L, so 1 000 of them are needed. The two arguments agree, and between them they are the reason a cubic metre is a thousand *litres* and a million *cubic centimetres* — the thousand counts litre cubes, not centimetre cubes.

P19 0.564 m

2 000 L is 2 m³, because the bridge from litres to cubic metres is ÷ 1 000. Then πr² × 2 = 2, so r² = 1 ÷ π and r = 0.564 m to 3 significant figures — a tank 1.13 m across. Check it forwards: π × 0.564² × 2 = 2.00 m³, which is 2 000 L.

P20 810 L

Capacity is a volume, and scaling every length by k multiplies a volume by **k³** — that is **G11** and **G18**, used here and not reproved. So the factor is 1.5³ = 3.375, and 240 × 3.375 = 810 L. A student who used 1.5 would get 360 L and a student who used 1.5² would get 540 L; the justification is that all three dimensions grow, so the factor appears three times.

P29 0.032π L, which is 0.101 L; and 0.8% full

The surface radius is 20 × 6 ÷ 30 = 4 cm, so the capacity is ⅓ × π × 16 × 6 = 32π cm³ = 0.032π L = 0.101 L. The percentage is (6 ÷ 30)³ = 0.2³ = 0.008, which is 0.8%. It is small because all three dimensions of the water have shrunk to a fifth, and a fifth cubed is a hundred and twenty-fifth — the first centimetres of depth in a cone are worth almost nothing.

P30 11.25 L

The drum has straight sides, so at half its depth it holds half its capacity: 15 L. The cone at half its depth holds (½)³ = one eighth of its capacity: 30 ÷ 8 = 3.75 L. The difference is 15 − 3.75 = 11.25 L. Two vessels of identical depth and identical capacity, filled to identical depths, holding four times different amounts — which is the whole point of this concept.

P39 51.2 min

Two stages, so two calculations. For the first 20 minutes the net rate is 25 − 4 = 21 L/min, which delivers 21 × 20 = 420 L. That leaves 1 200 − 420 = 780 L, now filling at the full 25 L/min, which takes 780 ÷ 25 = 31.2 min. The total is 20 + 31.2 = 51.2 min. Had the leak never been repaired it would have taken 1 200 ÷ 21 = 57.1 min.

P40 40.7 min

The capacity is ⅓ × π × 81 × 24 = 648π cm³ = 0.648π L = 2.04 L, so the time is 0.648π ÷ 0.05 = 40.7 min to 3 significant figures. Worth noticing: the vessel reaches half its *depth* after only 40.7 ÷ 8 = 5.09 minutes, because the bottom half of a cone holds an eighth of it. The water climbs fast at first and then crawls.

P49 the cone takes π min, and the whole vessel takes 5.5π min, which is 17.3 min

The cone holds 4π L, so at 4 L/min it takes exactly π minutes, about 3.14 min. The whole funnel holds 22π L, so it takes 22π ÷ 4 = 5.5π min = 17.3 min. The cone is 30 cm of the vessel's 75 cm and takes only 3.14 of its 17.3 minutes — the water rises quickly through the narrow part and then slows right down.

P50 2π L, which is exactly four thirds of the cylinder’s 1.5π L

The cylinder holds π × 100 × 15 = 1 500π cm³ = 1.5π L and the cone holds a third of that, 0.5π L, so the vessel holds 2π L — and 2 ÷ 1.5 = 4 ÷ 3 exactly. It is true for any radius and any height, provided the cone has the same radius and the same height as the cylinder: the capacity is $\pi r^2 h + \frac{1}{3}\pi r^2 h = (4 \div 3)\pi r^2 h$, and every r and every h cancels out of the ratio.

P59 0 significant figures — the bounds 4 690 L and 5 510 L do not agree even on their first digit

Work the two bounds and then round them together, one figure at a time, until they agree. To 1 significant figure they are 5 000 L and 6 000 L, which is already a disagreement, so **not one** figure of 5 090 L can be defended on its own. All that can honestly be said is that the tank holds somewhere between about 4 700 L and 5 500 L. Quoting 5 090 L claims a precision the tape never had. This is exactly **N18 Trapped Between Bounds**, applied to a capacity, and it is also why the 5 000 L on the label cannot be called wrong.

P60 12 drums, for 2 400 L

Cross the bridge first, and cross it by a thousand: 2.4 m³ = 2 400 L. Each drum really holds 66.64π L = 209 L, so 2 400 ÷ 209.36 = 11.5. Containers

are whole things and 11 drums would leave about 100 L on the ground, so the count rounds **up** to 12 — the opposite decision from P16, where a part bucketful was not a bucketful. The rounding follows the situation, not a rule.