Number · Unit N09

Building Blocks

Foundations · MYP 1–3 · 60 practice questions · Criteria A, B, C, D

Every whole number is built out of primes, and there is only one way to build it. Find those building blocks and a whole family of questions collapses into the same short piece of work: how many identical hampers can be packed from 96, 72 and 120 items, when two combis next leave the rank together, and whether 8316 divides exactly by 12.

By the end of this unit you can

• Use the tests for 2, 3, 4, 5, 6, 8, 9 and 10 to decide divisibility without dividing

• List every factor of a number in pairs, and explain why 1 is not a prime number

• Break a number into primes with a factor tree and write the answer in index form

• Read the HCF of two numbers from a Venn diagram of their prime factors

• Read the LCM from the same diagram, and check it with HCF × LCM = the product

• Decide from the wording of a problem whether it wants the HCF or the LCM, and defend the choice with a size test

The one idea behind all of it

Every whole number greater than 1 is a product of prime numbers, and there is exactly one way to write it. That one fact runs the whole unit. The prime factorisation tells you what divides the number, how many factors it has, what it shares with another number — the HCF — and what the two of them build together — the LCM.

Give every final answer to 3 significant figures, unless the answer is exact. An exact answer such as 25, 0.5 or 4 000 is written as it is, and the rounding is done once, at the very end.

360 = 2³ × 3² × 5 — one number, one factorisation. What two numbers share is the HCF; what they build together is the LCM.

Where this unit sits

This unit is about whole numbers only. It picks up where N01 *Know Your Numbers* left off: there you learned to calculate with integers, here you take them apart. Nothing in this unit needs a calculator, and nothing in it is a fraction.

It bridges forward to N10 Squares, Roots & Powers, where the index notation you meet here — the small raised number in 2^3 — becomes the subject in its own right. The full laws of indices belong to the Algebra course, unit 11 *The Power Rules*; the two courses share the notation and never repeat each other's work.

You have already used both of this unit's big ideas without naming them. Simplifying a fraction is dividing top and bottom by their HCF, and adding fractions with different denominators means finding their LCM — both from N02 *Fraction Fluency*. The two-circle Venn diagram drawn here is the same diagram the Geometry strand uses for sets.

Concept 1 · Tests that save you time

Divisibility by 2, 3, 4, 5, 6, 8, 9 and 10 — without dividing

Before you divide anything, look at it. A handful of tests decide divisibility by 2, 3, 4, 5, 6, 8, 9 and 10 in a couple of seconds, and they are what get a factor tree started. Two of them do nearly all the work: is it even, and does the digit sum go into 3?

Divisible by	The test	5148
2	the last digit is even (0, 2, 4, 6 or 8)	yes
3	the digit sum is a multiple of 3	yes
4	the last two digits form a multiple of 4	yes
5	the last digit is 0 or 5	no

Divisible by	The test	5148
6	it passes the test for 2 AND the test for 3	yes
8	the last three digits form a multiple of 8	no
9	the digit sum is a multiple of 9	yes
10	the last digit is 0	no

Worked example 1 · Test 5148 against all eight tests

1	Divisible by 2? The last digit is 8, which is even.	**yes**
2	Divisible by 3? The digit sum is 5 + 1 + 4 + 8 = 18, and 18 = 3 × 6.	**yes**
3	Divisible by 4? The last two digits are 48, and 48 = 4 × 12.	**yes**
4	Divisible by 5 or by 10? The last digit is 8, not 0 and not 5.	**no to both**
5	Divisible by 6? It passed the test for 2 and the test for 3.	**yes**
6	Divisible by 8? The last three digits are 148, and 148 ÷ 8 = 18.5.	**no**
7	Divisible by 9? The digit sum is 18, and 18 = 9 × 2.	**yes**

Every one of those answers is hiding inside the prime factorisation: $5148 = 2^2 \times 3^2 \times 11 \times 13$. There are two 2s, so 4 divides it but 8 does not. There are two 3s, so 9 divides it. There is no 5, so neither 5 nor 10 divides it. The tests are quick; the prime factorisation is the reason they work.

Concept 2 · Factors, multiples and primes

Listing every factor in pairs, and what makes a number prime

A factor divides in exactly; a multiple is what you get when you multiply. Factors come in pairs, so list them by walking up from 1 and writing both

members of each pair. A prime has exactly two different factors — which is why 1, with its single factor, is not prime.

Worked example 2 · Every factor of 48, and is 51 prime?

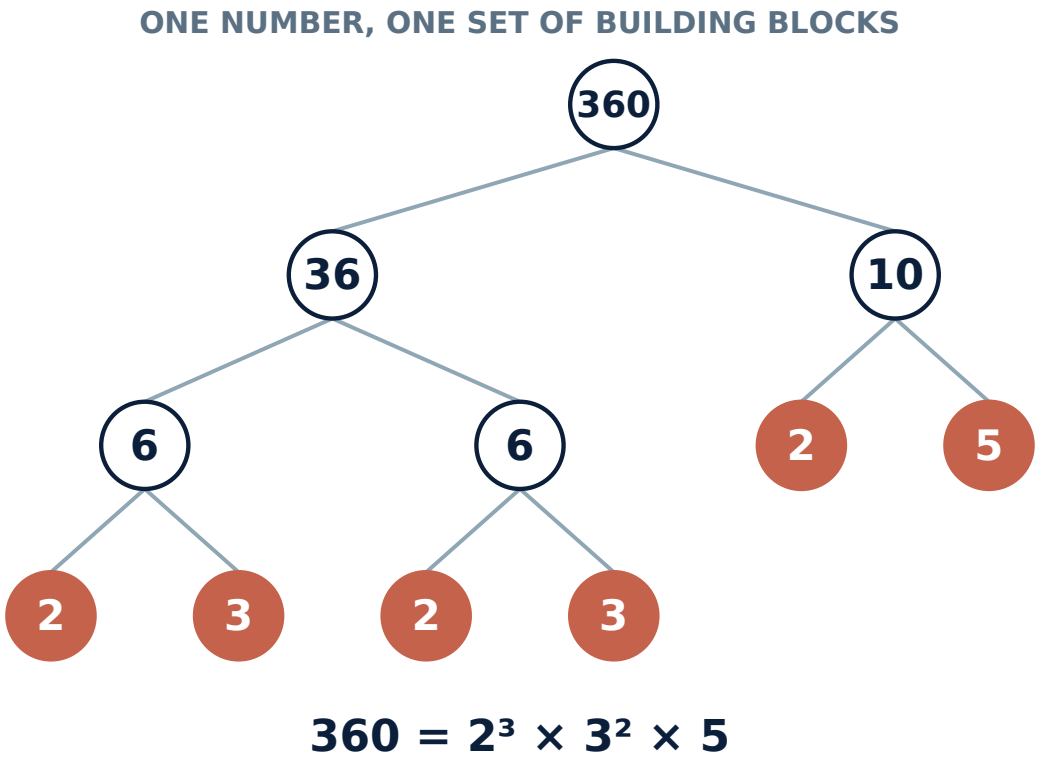
1	Start at 1 and work upwards in pairs. 1 × 48, 2 × 24, 3 × 16, 4 × 12.	**four pairs so far**
2	5 is not a factor (48 does not end in 0 or 5). 6 × 8 is the next pair.	**1, 2, 3, 4, 6, 8, 12, 16, 24, 48**
3	Stop at 7, because 7 × 7 = 49 is already past 48 — every pair has been found.	**10 factors**
4	Now 51: it is odd, and its digit sum is 5 + 1 = 6, a multiple of 3.	**3 divides it**
5	51 = 3 × 17, so 51 has the factors 1, 3, 17 and 51.	**51 is not prime**

Factors arrive in pairs, which is why most numbers have an even number of them. A square number is the exception: 36 = 6 × 6 uses the same factor twice, so 36 has nine factors, an odd number. Hold on to that — the Criterion B investigation below is built on it.

Concept 3 · Breaking a number into primes

The factor tree, and writing the answer in index form

A factor tree takes a number apart until every branch ends in a prime. Split it any way you can see: the tree looks different each time and the circled primes never change. Collect equal primes into powers and you have the index form.



$$360 = 2^3 \times 3^2 \times 5$$

Split 360 any way you like: the branches differ, the six circled primes never do. Primes are shown in the filled circles — a branch stops only when it reaches one.

Worked example 3 · Factor tree of 360, and the index form

1	Choose any split you can see. 360 = 36 × 10.	**two branches**
2	Neither is prime, so split again: 36 = 6 × 6 and 10 = 2 × 5.	**2 and 5 are prime**
3	6 is still not prime: 6 = 2 × 3, twice over.	**every branch now ends**
4	Circle the leaves and write them in order.	**2 × 2 × 2 × 3 × 3 × 5**
5	Collect equal primes into powers.	**360 = $2^3 \times 3^2 \times 5$**
6	Check by multiplying back: 8 × 9 × 5.	**360** ✓

The first split does not matter. Starting 360 = 4 × 90 or 360 = 2 × 180 gives a different-looking tree and exactly the same six primes. Every whole number bigger than 1 has one and only one prime factorisation — that is why it is worth finding.

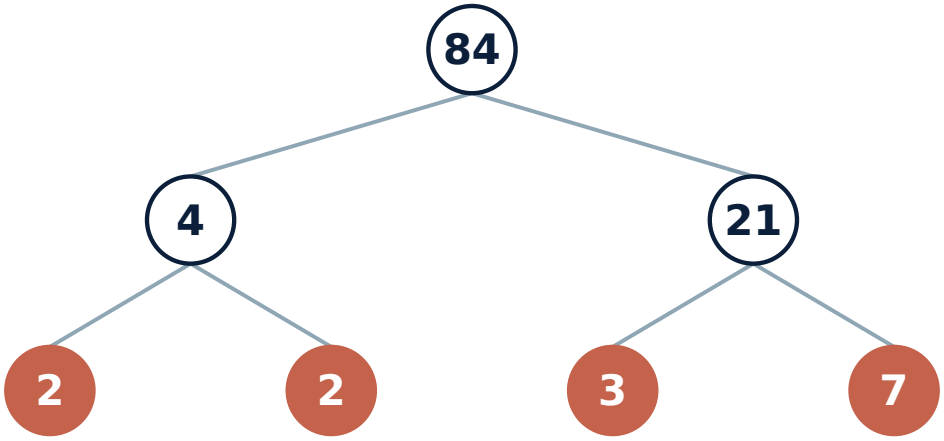
Concept 4 · The highest common factor

Two circles, one overlap — the HCF is what they share

The highest common factor is the biggest number that divides into both. Break each number into primes, draw two circles, and put every shared prime in the overlap. Multiply the overlap and you have the HCF. An empty overlap means the numbers are co-prime and the HCF is 1.

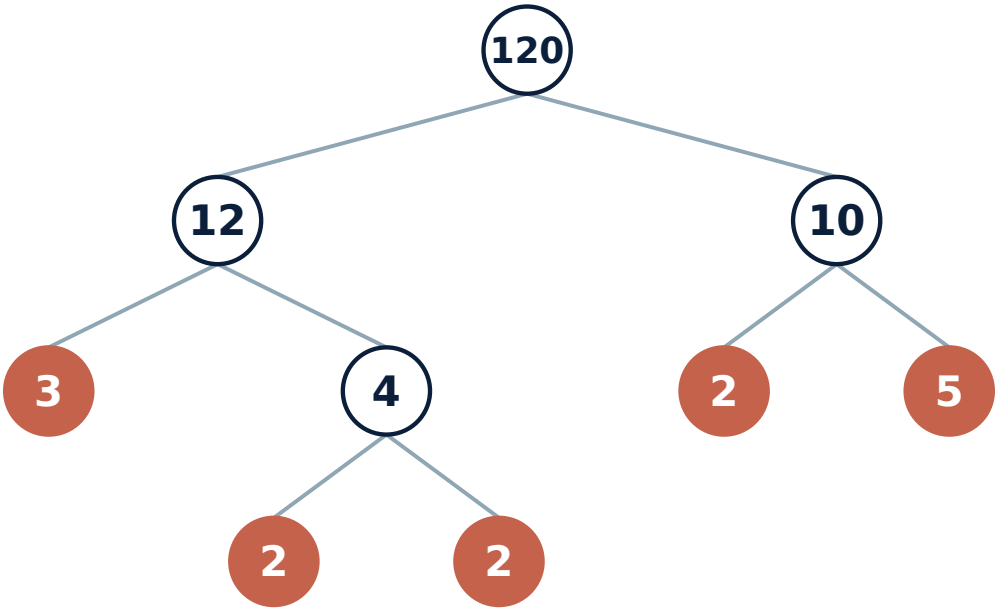
	What you take	84 = $2^2 \times 3 \times 7$, 120 = $2^3 \times 3 \times 5$
HCF	the lower power of every shared prime	$2^2 \times 3 = 12$
LCM	the higher power of every prime that appears	$2^3 \times 3 \times 5 \times 7 = 840$

THE FACTOR TREE FOR 84



$84 = 2^2 \times 3 \times 7$

THE FACTOR TREE FOR 120

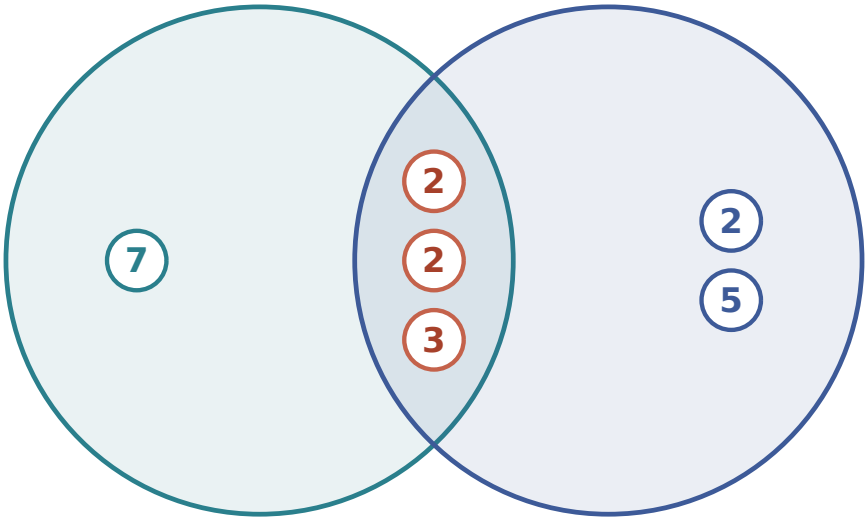


$120 = 2^3 \times 3 \times 5$

Two trees, two index forms — everything the Venn diagram below needs.

THE PRIME FACTORS OF 84 AND 120

$84 = 2^2 \times 3 \times 7$ **$120 = 2^3 \times 3 \times 5$**



HCF = the overlap = 2 × 2 × 3 = 12

LCM = the overlap × the rest = 12 × 7 × 2 × 5 = 840

The overlap is the HCF. The whole diagram, taking each circle's leftovers once, is the LCM. One picture answers both questions.

Worked example 4 · HCF of 84 and 120 from a Venn diagram

1	Break each number into primes.	**$84 = 2^2 \times 3 \times 7$**
2	And the other one.	**$120 = 2^3 \times 3 \times 5$**
3	Put the shared primes in the overlap: one 2, a second 2, and a 3. The 7 is left in the 84 circle; a third 2 and the 5 are left in the 120 circle.	**overlap = 2, 2, 3**
4	Multiply the overlap.	**HCF = 2 × 2 × 3 = 12**
5	Check: 84 ÷ 12 = 7 and 120 ÷ 12 = 10, and 7 and 10 share no factor.	**12 cannot be beaten**

The overlap is the HCF, always. If the two circles share nothing, the overlap is empty and the HCF is 1 — the numbers are then called co-prime.

Concept 5 · The lowest common multiple

The same diagram, read the other way — everything, once each

The lowest common multiple is the smallest number they both divide into. It is the same diagram read whole: the overlap once, plus whatever is left in each circle. Always check with HCF × LCM = the product of the two numbers.

Worked example 5 · LCM of 84 and 120 from the same diagram

1	Do not redraw anything. Read the whole diagram this time.	**overlap 2, 2, 3 · left 7 · right 2, 5**
2	Multiply everything in the diagram, taking each circle's leftovers once.	**LCM = 12 × 7 × 2 × 5**
3	Work it out.	**LCM = 840**
4	Check both ways round: 840 ÷ 84 and 840 ÷ 120.	**10 and 7** ✓
5	Check the pairing rule: HCF × LCM = 12 × 840, and 84 × 120.	**10080 = 10080** ✓

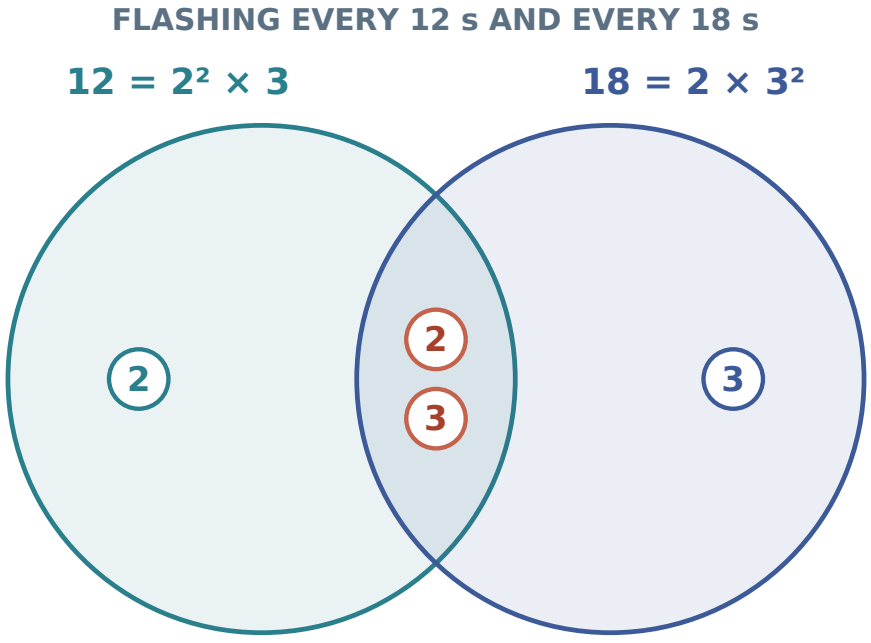
One diagram, two answers. The overlap alone is the HCF; the whole diagram is the LCM. And HCF × LCM = the product of the two numbers, every time — a free check you should use whenever you have found both.

Concept 6 · Which one does the question want?

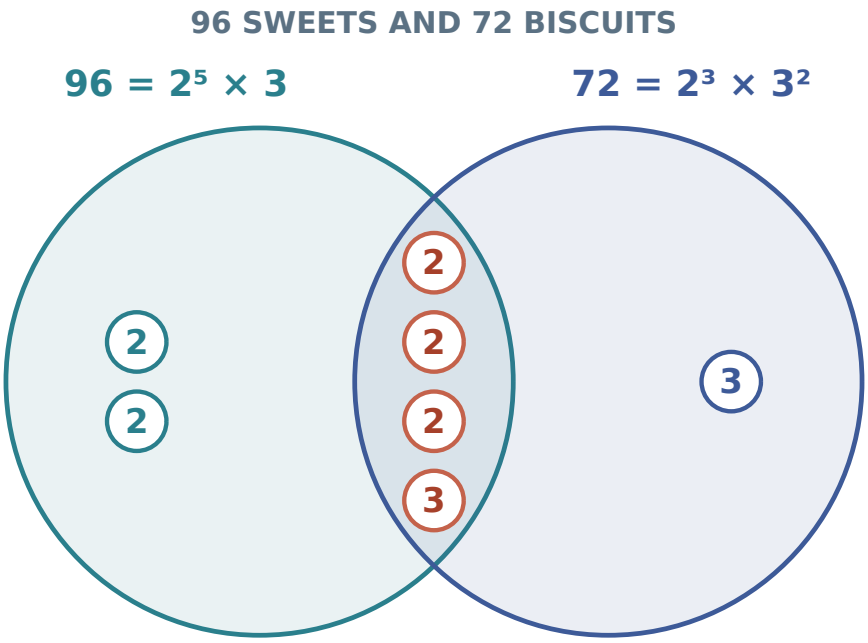
Packing, tiling and syncing — choosing HCF or LCM and defending it

In a word problem nobody tells you which one to use. Sort it first: are you cutting something up, or bringing things back together? Then size-check the answer against the numbers in the question.

The question sounds like	You need	Size check
equal bags, equal groups, sharing out, the largest tile, cutting into equal pieces, the biggest spacing	HCF	never bigger than the smallest number given
flashing together, meeting again, the next time both, laps of a track, the smallest number both divide into	LCM	never smaller than the biggest number given



The lights flash together every 36 seconds — the LCM. The HCF of 6 seconds is also on the diagram, but it answers a different question.



HCF = the overlap = 2 × 2 × 2 × 3 = 24

LCM = the overlap × the rest = 24 × 2 × 2 × 3 = 288

Here it is the overlap that matters: 24 identical bags, each holding 96 ÷ 24 = 4 sweets and 72 ÷ 24 = 3 biscuits.

Worked example 6 · Two lights, 12 seconds and 18 seconds

1	Read the question: they start together and we want the NEXT time together, so the answer must be bigger than both numbers.	**this is an LCM**
2	Break both into primes.	**12 = 2² × 3 · 18 = 2 × 3²**
3	Take the higher power of each prime.	**2² × 3² = 36**
4	Check by listing: 12, 24, 36 and 18, 36.	**36 s is the first match**
5	In a 3-minute advert, 180 ÷ 36.	**5 times together**

The words to hunt for are "together again", "next time" and "smallest number that both divide". All three mean LCM, and all three have an answer at least as big as the largest number in the question.

Worked example 7 · Equal gift bags from 96 sweets and 72 biscuits

1	Read the question: the bags are cut out of what is already there, so the answer cannot be bigger than 72.	**this is an HCF**
2	Break both into primes.	**$96 = 2^5 \times 3 \cdot 72 = 2^3 \times 3^2$**
3	Take the lower power of each shared prime.	**$2^3 \times 3 = 24$**
4	Share out what each bag gets: 96 ÷ 24 and 72 ÷ 24.	**4 sweets, 3 biscuits**
5	Check nothing is left over: 24 × 4 and 24 × 3.	**96 and 72 ✓**

Same two numbers, opposite question. The LCM of 96 and 72 is 288, which would be more bags than you have sweets — an answer that fails the common-sense test before you even check the arithmetic.

The building-blocks routine

1. Run the quick tests first — Is it even? Does the digit sum go into 3? Does it end in 0 or 5? Three glances usually give you the first split of the tree, and they cost nothing.

2. Build the tree and circle every prime — Any first split works. Keep splitting until every branch ends in a circled prime — a leaf you have not circled is a branch you have not finished.

3. Write it in index form, primes in order — $2 \times 2 \times 2 \times 3 \times 3 \times 5$ becomes $2^3 \times 3^2 \times 5$. Now the number is in the form every later question wants.

✓ **Ask which answer the question wants** — Cutting up, packing, sharing, largest tile → HCF, and the answer is never bigger than the smallest number given. Meeting again, flashing together, smallest number both divide → LCM, and the answer is never smaller than the biggest number given.

The mistakes that cost the most marks

Trap 1 · 1 is counted as a prime number

WRONG "The factors of 12 are 1, 2, 3, 4, 6 and 12, so its prime factors are 1, 2 and 3, and $12 = 1 \times 2^2 \times 3$."

RIGHT A prime number has **exactly two different factors**. 1 has only one factor — itself — so 1 is not prime (and it is not composite either). $12 = 2^2 \times 3$.

If 1 were allowed, 12 could also be written $1 \times 2^2 \times 3$, or $1^2 \times 2^2 \times 3$, or $1^{99} \times 2^2 \times 3$, and a number would no longer have exactly one prime factorisation. That is the real reason 1 is excluded. While you are here: 2 *is* prime — it is the only even prime, because every other even number has 2 as a third factor.

Trap 2 · HCF and LCM swapped in a word problem

WRONG "96 sweets and 72 biscuits, so the number of bags is the LCM, 288."

RIGHT The bags are made out of what you already have, so the answer must be **smaller** than 72. It is the HCF: 24 bags.

Sort the question before you calculate. **Cutting up, sharing out, packing into equal groups, the largest tile** → HCF, and the answer is never bigger than the smallest number in the question. **Meeting again, flashing together, the smallest number both divide into** → LCM, and the answer is never smaller than the biggest number in the question. Check your answer against that size test every time.

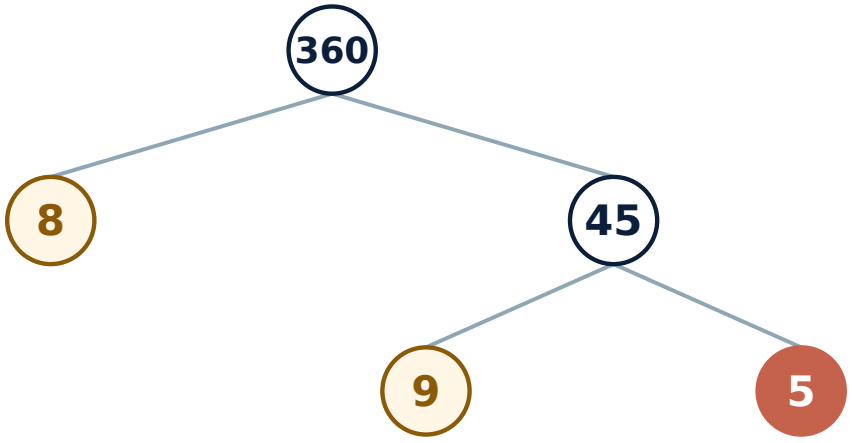
Trap 3 · The factor tree is stopped before every leaf is prime

WRONG "360 = 8 × 45 = 8 × 9 × 5, so the prime factorisation is 8 × 9 × 5."

RIGHT 8 and 9 are not prime, so the tree is not finished. 8 = 2 × 2 × 2 and 9 = 3 × 3, giving 360 = $2^3 \times 3^2 \times 5$.

Before you stop, run each leaf past the tests: is it even? is its digit sum a multiple of 3? does it end in 0 or 5? A leaf that answers yes to any of those can still be split. Circle every prime as you finish it — an uncircled leaf is an unfinished branch.

THE TREE THAT WAS STOPPED TOO EARLY



8 and 9 are not prime, so this tree is not finished

The two pale leaves are still composite: 8 = 2 × 2 × 2 and 9 = 3 × 3. Finish them and 360 = $2^3 \times 3^2 \times 5$.

Practice · Fluency · 24 questions

P1 **Write down** whether 4218 is divisible by 2.

P2 **Write down** whether 5431 is divisible by 3.

P3 **Write down** whether 7320 is divisible by 5 and by 10.

P4 **Write down** whether 9144 is divisible by 4.

P11 **Write down** all the factors of 18.

P12 **Write down** the first four multiples of 7.

P13 **Write down** whether 1 is a prime number.

P14 **Write down** the prime numbers between 20 and 30.

P21 **Write down** 24 as a product of primes in index form.

P22 **Write down** 45 as a product of primes in index form.

P23 **Write down** the value of $2^2 \times 5^3$.

P24 **Write down** the prime factorisation of 100 in index form.

P31 **Write down** the HCF of 8 and 12.

P32 **Write down** the HCF of 9 and 15.

P33 **Write down** the HCF of 7 and 13.

P34 **Write down** the HCF of 6 and 24.

P41 **Write down** the LCM of 4 and 6.

P42 **Write down** the LCM of 5 and 8.

P43 **Write down** the LCM of 3 and 12.

P44 **Write down** the LCM of 10 and 15.

P51 Two combis leave the rank at the same moment and then run on fixed intervals. **Write down** whether you need the HCF or the LCM to find when they next leave together.

P52 Two ribbons are to be cut into equal pieces that are as long as possible, with nothing left over. **Write down** whether you need the HCF or the LCM.

P53 Two lights flash every 4 seconds and every 6 seconds. They flash together now. **Write down** after how many seconds they next flash together.

P54 **Write down** the largest number of identical party bags that can be made from 12 pens and 18 notebooks with nothing left over.

Practice · Applied · 24 questions

P5 **Determine** whether 3852 is divisible by 6.

P6 **Determine** whether 5184 is divisible by 8.

P7 **Determine** whether 62613 is divisible by 9.

P8 In the number 47*d*2 the digit *d* is missing. **Determine** the smallest digit *d* that makes the number divisible by 3.

P15 **Determine** all the factors of 48 by working through the factor pairs.

P16 **Determine** whether 91 is a prime number.

P17 **Determine** how many factors 36 has.

P18 **Determine** the largest prime number below 100.

P25 **Determine** the prime factorisation of 360 using a factor tree.

P26 **Determine** the prime factorisation of 588.

P27 **Determine** the prime factorisation of 1000 in index form.

P28 **Determine** the number whose prime factorisation is $2^4 \times 3 \times 5^2$.

P35 **Determine** the HCF of 84 and 120 using prime factors.

P36 **Determine** the HCF of 90 and 126.

P37 **Determine** the HCF of 45, 60 and 75.

P38 Two numbers have prime factorisations $2^3 \times 3^2 \times 5$ and $2^2 \times 3 \times 7$. **Determine** their HCF, leaving your answer in index form.

P45 **Determine** the LCM of 84 and 120 using prime factors.

P46 **Determine** the LCM of 12 and 18.

P47 Two numbers have prime factorisations $2^3 \times 3$ and $2 \times 3^2 \times 5$. **Determine** their LCM.

P48 **Determine** the LCM of 8, 12 and 20.

P55 One light flashes every 12 seconds and another every 18 seconds. They flash together at 07:00. **Determine** after how many seconds they next flash together.

P56 A shop has 96 sweets and 72 biscuits. Identical bags are made using all of them. **Determine** the largest number of bags, and what goes into each bag.

P57 Combis on one route leave a rank every 15 minutes and on another route every 24 minutes. Both leave at 06:00. **Determine** the next time both routes leave together.

P58 A floor 96 cm by 72 cm is to be covered exactly by identical square tiles, with no tile cut. **Determine** the largest tile that can be used and how many tiles are needed.

Practice · Challenge · 12 questions

P9 A number is divisible by 12 exactly when it is divisible by both 4 and 3. **Determine** whether 8316 is divisible by 12, showing both tests.

P10 **Determine** a number that is divisible by both 4 and 6 but is *not* divisible by 24, and **describe** why such a number exists.

P19 **Determine** every number below 50 that has exactly three factors, and **describe** what they have in common.

P20 A number has exactly two factors and is even. **Determine** the number.

P29 **Determine** the prime factorisation of 1752.

P30 A number has prime factorisation $2^a \times 3$. **Determine** the smallest value of a for which the number is greater than 100.

P39 **Determine** the highest common factor of 1752 and 356.

P40 The HCF of two numbers is 6, and one of the numbers is 24. **Determine** the smallest the other number could be.

P49 Two numbers have HCF 12 and LCM 840. One of the numbers is 84. **Determine** the other number.

P50 **Determine** the smallest number greater than 1 that leaves a remainder of 1 when it is divided by 4, by 5 and by 6.

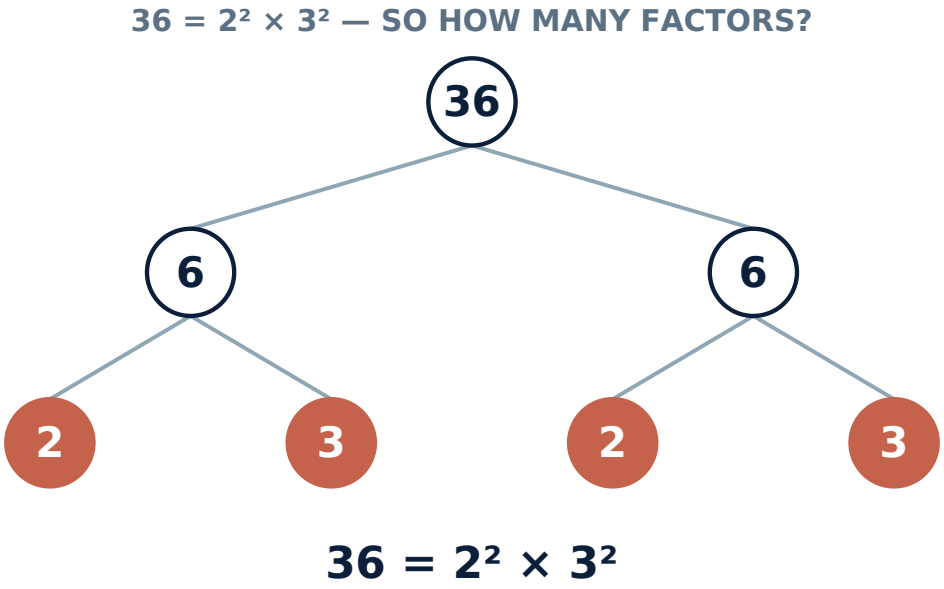
P59 Three routes leave a rank every 15, 20 and 24 minutes, all together at 06:00. **Determine** the next time all three leave together, and **describe** one reason the real rank might not match your answer.

P60 A rectangular plot is 132 m by 84 m. Fence posts are placed all the way round, equally spaced, with a post at each corner. **Determine** the largest possible spacing and the number of posts needed.

Criterion B · Investigating patterns

How many factors does a number have?

Counting the factors of a number by listing them is slow, and it is easy to miss one. There is a much faster way, and it is hiding in the index form. This investigation asks you to find it.



Two 2s and two 3s. The Criterion B investigation asks you to find what the indices 2 and 2 have to do with the nine factors of 36.

Part 1 · structured

n	Every factor of n	How many	Prime factorisation
6	1, 2, 3, 6	4	2 × 3
8	1, 2, 4, 8	4	2^3
12			
16			
18			
20			
24			
36			

1. **Complete** the table. Take your time over the factor lists — the whole investigation depends on them being right, so pair them off as in worked example 2.

2. **Predict** how many factors 48 has, and how many 100 has, before you list either of them. Write both predictions down first, then check them by listing.

3. **Identify** what is going on. Compare the indices in the last column with the number in the third column, row by row. Say in words what you notice, first for the numbers built from one prime and then for the numbers built from two.

4. **Determine** a general rule for the number of factors of $n = p^a \times q^b$, where p and q are different primes. Your rule must work for any a and any b, and it must say what to do when there is only one prime.

5. **Test and verify** your rule on 84 and on 200, by using the rule and then by listing every factor the long way. The two answers must agree, and you should show both.

6. **Justify** why the rule works. Think about how a factor of n is built out of the primes of n: how many 2s could it take from 2^3, and how many 3s from 3^2? Explain why that counting gives your rule, and why the +1 is there.

Part 2 · open

Now you choose what to investigate, and how. Take one of these openings or propose your own. You decide which numbers to try, how many you need, how to record them and how you will test whatever you find.

1. Which numbers have **exactly three** factors? **Investigate**, find them all below 200, and use your rule from Part 1 to say exactly what kind of number they are — and why there can be no others.

2. Which numbers have an **odd** number of factors? **Investigate**. The answer is a famous family of numbers, and worked example 2 has already dropped a hint. Explain what makes them different, in terms of factor pairs.

3. Which number below 100 has the **most** factors? **Investigate**, and then **determine** whether the biggest numbers are the ones with the most factors — or whether something else decides it.

Whichever you choose, take it through the same ladder: **predict**, **identify** the pattern, **determine** a general rule, **test and verify** it on cases you have not already used, and **justify** why it must hold and where it stops holding.

Marked out of 8 against Criterion B. Completing the table earns the lower bands; the general rule, the verification and the justification are what reach levels 5–8. Part 2 must show your own choice of method.

Remember, you should communicate in an organized and coherent manner.

Criterion D · Applying maths in real-life contexts

Hampers, combis and a Saturday at the market

Mma Kelebogile Selepeng runs a stall at the Broadhurst market in Gaborone. On the last Saturday of the month she wants to make up identical hampers from stock she already has, and she needs to know when the two combi routes her customers use leave the rank together, so that she can be standing at the stall when they arrive. **Both tables below contain more information than you need.**

Source 1 · the stock list

Item	In stock	Unit cost	Supplier	Shelf
Bars of soap	96	P6.50	Gaborone Wholesalers	A2
Packets of sugar (2 kg)	72	P12.95	Gaborone Wholesalers	A4
Boxes of tea (100 bags)	120	P8.75	Kgale Traders	B1
Tins of jam	45	P14.20	Kgale Traders	B3
Packets of biscuits	17	P9.60	Riverwalk Suppliers	C1
Bottles of cooking oil (750 ml)	30	P22.40	Kgale Traders	C2
Bags of maize meal (5 kg)	54	P39.95	Gaborone Wholesalers	D1

She has decided that every hamper will contain soap, sugar and tea, and that no item of those three may be left over. She sells each hamper for P135.00. Prices are in pula and thebe; names, quantities and prices are illustrative and were written for this task.

Source 2 · the combi rank board

Route	Where it goes	Minutes between departures	Fare	Distance
Route 1	Gaborone rank → Mogoditshane	15	P5.50	12 km
Route 2	Gaborone rank → Tlokweng	24	P6.00	15 km
Route 3	Gaborone rank → Mochudi	40	P12.50	37 km
Route 4	Gaborone rank → Gabane	20	P7.00	18 km
Route 5	Gaborone rank → Ramotswa	45	P14.00	30 km

Her customers use Route 1 and Route 2. Every route sends its first combi at 06:00 and its last at 18:00. The board assumes each route keeps to its stated interval exactly.

Identify the information you actually need from the two tables, and **describe** why each column you are leaving out does not matter for these two questions. *(Strand i — identifying the relevant elements)*

Select your strategies. **Write down** which of the two questions needs the HCF and which needs the LCM, and say how the wording told you. Then write each quantity you need as a product of primes. *(Strand ii — selecting appropriate strategies)*

Apply them. **Determine** the largest number of identical hampers she can make, what goes into one hamper, what one hamper costs her (money to two decimal places) and what she makes on the whole batch. Then **determine** the next time after 06:00 that both routes leave together, and how many times they do so before the rank closes. *(Strand iii — applying the strategy)*

Describe how accurate your two answers can honestly be. The stock counts are exact whole numbers, so what does that mean for the hamper answer? The departure intervals are not: at a real rank a combi usually leaves when it is full, not when the clock says so. **Describe** what that does to your 08:00 answer, and **describe** what rounding money to two decimal places means when you multiply a single hamper's profit by the whole batch. *(Strand iv — accuracy of the solution)*

Describe whether your answers make sense at a real market stall. She also has 17 packets of biscuits. **Determine** what happens to the number of hampers if she insists on using all of those too, and **describe** why. Then **describe** what she should do with the stock that is left over on the shelf, and whether making the largest possible number of hampers is

actually the best decision for one Saturday. *(Strand v — does it make sense in context)*

Submit: your prime factorisations, your two answers with working, your money to two decimal places, and short written answers for parts 1, 4 and 5. **Marked out of 8** against Criterion D. Strands iv and v carry as much weight as the calculations.

Answers

How the P-numbers run. The 60 practice items are numbered P1 to P60 in the order they appear in the lesson — ten per concept, and inside each concept four Fluency, then four Applied, then two Challenge. The same P-number identifies the item in the lesson, in this key and in the teacher pack, so the numbers inside each tier below step in blocks of ten rather than running consecutively: Fluency is P1–P4, P11–P14, P21–P24 and so on; Applied is P5–P8, P15–P18, …; Challenge is P9–P10, P19–P20, …

Fluency · F-tier (24)

P1	**yes**
P2	**no**
P3	**yes to both**
P4	**yes**
P11	**1, 2, 3, 6, 9, 18**
P12	**7, 14, 21, 28**
P13	**no**
P14	**23, 29**
P21	**$2^3 \times 3$**
P22	**$3^2 \times 5$**
P23	**500**
P24	**$2^2 \times 5^2$**
P31	**4**
P32	**3**
P33	**1**
P34	**6**
P41	**12**

P42	**40**
P43	**12**
P44	**30**
P51	**the LCM**
P52	**the HCF**
P53	**12 seconds**
P54	**6**

Applied — fully worked (24)

P5 yes

6 = 2 × 3, so run both tests. Last digit 2 is even, so it passes the test for 2. Digit sum 3 + 8 + 5 + 2 = 18, and 18 = 3 × 6, so it passes the test for 3. It passes both, so it is divisible by 6. Check: 3852 ÷ 6 = 642.

P6 yes

Test for 8: look at the last three digits, 184. Since 184 = 8 × 23, the whole number is divisible by 8. Check: 5184 ÷ 8 = 648.

P7 yes

Digit sum 6 + 2 + 6 + 1 + 3 = 18, and 18 = 9 × 2, so it is divisible by 9. Note it is not divisible by 2, because the last digit is odd. Check: 62613 ÷ 9 = 6957.

P8 2

The digit sum is 4 + 7 + d + 2 = 13 + d. For divisibility by 3 the digit sum must be a multiple of 3, so 13 + d = 15, 18 or 21, giving d = 2, 5 or 8. The smallest is d = 2, and 4722 ÷ 3 = 1574.

P15 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Work upwards in pairs: 1 × 48, 2 × 24, 3 × 16, 4 × 12, 6 × 8. 5 is not a factor. Stop at 7, because 7 × 7 = 49 is already past 48, so every pair has been found. That is 10 factors.

P16 no, 91 = 7 × 13

Test the primes in turn: 91 is odd, its digit sum is 10 so 3 fails, it does not end in 0 or 5, and 91 ÷ 7 = 13 exactly. Since 91 has a third factor it is not

prime. You only ever have to test primes up to the square root: $9^2 = 81$ and $10^2 = 100$, so testing 2, 3, 5 and 7 was enough.

P17 9

Pair them off: 1×36, 2×18, 3×12, 4×9, 6×6. The last pair is a repeat, so 6 is counted once: 1, 2, 3, 4, 6, 9, 12, 18, 36 — nine factors.

P18 97

Work down from 99: 99 has digit sum 18, so 3 divides it; 98 is even; 97 is odd, has digit sum 16, does not end in 0 or 5, and $97 \div 7 = 13.857\ldots$. Since $10^2 = 100 > 97$, testing 2, 3, 5 and 7 is enough, so 97 is prime.

P25 $2^3 \times 3^2 \times 5$

Split it any way you like and keep going until every branch ends in a prime: $360 = 36 \times 10$, then $36 = 6 \times 6$ and $10 = 2 \times 5$, then each $6 = 2 \times 3$. The circled primes are 2, 3, 2, 3, 2, 5. In order: $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$.

P26 $2^2 \times 3 \times 7^2$

588 is even: $588 = 2 \times 294$, and $294 = 2 \times 147$. Now 147 has digit sum 12, so $147 = 3 \times 49$, and $49 = 7 \times 7$. Altogether $588 = 2 \times 2 \times 3 \times 7 \times 7 = 2^2 \times 3 \times 7^2$.

P27 $2^3 \times 5^3$

$1000 = 10 \times 10 \times 10$ and $10 = 2 \times 5$, so $1000 = (2 \times 5)^3 = 2^3 \times 5^3$. Every power of 10 has equal numbers of 2s and 5s.

P28 1200

$2^4 = 16$ and $5^2 = 25$, so the number is $16 \times 3 \times 25 = 48 \times 25 = 1200$.

P35 12

$84 = 2^2 \times 3 \times 7$ and $120 = 2^3 \times 3 \times 5$. Take the lower power of every prime they share: 2^2 and 3. The HCF is $2^2 \times 3 = 12$. Check: $84 \div 12 = 7$ and $120 \div 12 = 10$, and 7 and 10 share nothing, so nothing bigger will work.

P36 18

$90 = 2 \times 3^2 \times 5$ and $126 = 2 \times 3^2 \times 7$. The shared part is $2 \times 3^2 = 18$. Check: $90 \div 18 = 5$ and $126 \div 18 = 7$.

P37 15

$45 = 3^2 \times 5$, $60 = 2^2 \times 3 \times 5$ and $75 = 3 \times 5^2$. Only 3 and 5 appear in all three, and the lowest power of each is 3^1 and 5^1, so the HCF is $3 \times 5 = 15$. There is no 2 in the HCF because 45 and 75 are odd.

P38 $2^2 \times 3$, which is 12

Compare prime by prime and take the lower power each time: for 2 that is 2^2, for 3 it is 3^1, and 5 and 7 are missing from one number each so they are not in the HCF. The HCF is $2^2 \times 3 = 12$. You never need the numbers themselves — 360 and 84 — to do this.

P45 840

$84 = 2^2 \times 3 \times 7$ and $120 = 2^3 \times 3 \times 5$. Take the higher power of every prime that appears: 2^3, 3, 5 and 7. The LCM is $8 \times 3 \times 5 \times 7 = 840$. Check: $840 \div 84 = 10$ and $840 \div 120 = 7$.

P46 36

$12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. Higher power of each prime: 2^2 and 3^2. The LCM is $4 \times 9 = 36$. Check: $36 \div 12 = 3$ and $36 \div 18 = 2$.

P47 $2^3 \times 3^2 \times 5$, which is 360

Take the higher power of each prime: 2^3 from the first, 3^2 and 5 from the second. The LCM is $8 \times 9 \times 5 = 360$. The two numbers are 24 and 90, and $360 \div 24 = 15$ while $360 \div 90 = 4$.

P48 120

$8 = 2^3$, $12 = 2^2 \times 3$ and $20 = 2^2 \times 5$. The highest power of 2 anywhere is 2^3, and 3 and 5 each appear once, so the LCM is $8 \times 3 \times 5 = 120$. Check: $120 \div 8 = 15$, $120 \div 12 = 10$ and $120 \div 20 = 6$.

P55 36 seconds

This is a "when do they meet again" question, so it is the LCM. $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$, so the LCM is $2^2 \times 3^2 = 36$. The first light flashes at 12, 24, 36 s and the second at 18, 36 s — 36 s is the first time both appear.

P56 24 bags, each with 4 sweets and 3 biscuits

The bags are cut out of what is already there, so it is the HCF. $96 = 2^5 \times 3$ and $72 = 2^3 \times 3^2$, so the HCF is $2^3 \times 3 = 24$. Each bag then holds $96 \div 24 = 4$ sweets and $72 \div 24 = 3$ biscuits. Check: $24 \times 4 = 96$ and $24 \times 3 = 72$.

P57 08:00

It is the LCM. $15 = 3 \times 5$ and $24 = 2^3 \times 3$, so the LCM is $2^3 \times 3 \times 5 = 120$ minutes, which is 2 hours. 06:00 + 2 hours = 08:00.

P58 24 cm tiles, and 12 of them

The tile has to divide both 96 and 72, so the largest is the HCF, 24 cm. The floor is then $96 \div 24 = 4$ tiles one way and $72 \div 24 = 3$ tiles the other, so $4 \times 3 = 12$ tiles. Check: $12 \times 24^2 = 6912$ cm², and $96 \times 72 = 6912$ cm².

Challenge — fully worked (12)

P9 yes

Test for 4: the last two digits are 16, and 16 = 4 × 4, so it passes. Test for 3: the digit sum is 8 + 3 + 1 + 6 = 18, a multiple of 3, so it passes. It passes both, so it is divisible by 12. Check: 8316 ÷ 12 = 693. The pair 4 and 3 works because 4 and 3 share no prime factor.

P10 12

12 is divisible by 4 and by 6, but 12 ÷ 24 is not a whole number. The reason is that 4 and 6 share the prime factor 2: $4 = 2^2$ and $6 = 2 \times 3$, so the smallest number divisible by both is the LCM, $2^2 \times 3 = 12$, not $4 \times 6 = 24$. Multiplying the two tests together only works when the two numbers share no prime factor.

P19 4, 9, 25 and 49

4 has factors 1, 2, 4; 9 has 1, 3, 9; 25 has 1, 5, 25; 49 has 1, 7, 49. Each one is a prime squared — 2^2, 3^2, 5^2, 7^2. A number p^2 has exactly the three factors 1, p and p^2, and no other kind of number can have exactly three.

P20 2

Exactly two factors means the number is prime. Every even number bigger than 2 has 2 as a third factor, so it cannot be prime. That leaves 2 itself, whose only factors are 1 and 2. 2 is the only even prime number.

P29 $2^3 \times 3 \times 73$

1752 is even three times over: 1752 = 2 × 876 = 2 × 2 × 438 = 2 × 2 × 2 × 219. Now 219 has digit sum 12, so 219 = 3 × 73, and 73 is prime (test 2, 3, 5 and 7 — none divides it, and $9^2 = 81 > 73$). So $1752 = 2^3 \times 3 \times 73$.

P30 6

Build the list: $2^1 \times 3 = 6$, then 12, 24, 48, 96, 192. $2^5 \times 3 = 96$ is still below 100, and $2^6 \times 3 = 192$ is above it, so a = 6.

P39 4

$1752 = 2^3 \times 3 \times 73$ and $356 = 2^2 \times 89$. The only shared prime is 2, and the lower power is 2^2, so the HCF is 4. Check: 1752 ÷ 4 = 438 and 356 ÷ 4 = 89, and 89 is prime and is not a factor of 438, so 4 cannot be beaten.

P40 6

The other number must be a multiple of 6, so the candidates are 6, 12, 18, 24, …. HCF(24, 6) = 6, so 6 works and nothing smaller is a multiple of 6. Note 12 would give HCF(24, 12) = 12, which is too big, so bigger

candidates are not automatically better.

P49 120

For any two numbers, HCF × LCM = the product of the numbers. So 12 × 840 = 84 × the other number, giving 10080 = 84 × n and n = 10080 ÷ 84 = 120. Check: HCF(84, 120) = 12 and LCM(84, 120) = 840.

P50 61

A number that leaves remainder 1 is one more than a common multiple. The smallest common multiple of 4, 5 and 6 is the LCM: $4 = 2^2$, $5 = 5$, $6 = 2 \times 3$, so the LCM is $2^2 \times 3 \times 5 = 60$. The number is 60 + 1 = 61. Check: $61 = 4 \times 15 + 1 = 5 \times 12 + 1 = 6 \times 10 + 1$.

P59 08:00

$15 = 3 \times 5$, $20 = 2^2 \times 5$ and $24 = 2^3 \times 3$. The LCM takes the highest power of each: $2^3 \times 3 \times 5 = 120$ minutes = 2 hours, so 08:00. In a real rank a combi usually leaves when it is full rather than on a fixed interval, so the model is an idealisation and the true times will drift.

P60 12 m apart, and 36 posts

For a post to land on every corner the spacing must divide both 132 and 84, so the largest is the HCF. $132 = 2^2 \times 3 \times 11$ and $84 = 2^2 \times 3 \times 7$, so the HCF is $2^2 \times 3 = 12$ m. The perimeter is $2 \times (132 + 84) = 432$ m, and posts every 12 m round a closed loop need 432 ÷ 12 = 36 posts (going right round, the last post is the first one again, so there is no +1 here).