Number · Unit N18 · Extended

Trapped Between Bounds

Extended · MYP 4–5 · 60 practice questions · Criteria A, B, C, D

Coming from Unit 17 Accuracy on Trial and Unit 4 Round It Right: you already know that a rounded figure stands for an interval one rounding unit wide — this unit makes that interval formal, gives its two ends names, and calculates with them.

A rounded measurement is not a number, it is a trap with two walls. This unit finds both walls exactly, writes them as an inequality, draws them on a number line, and then does the hard part: pushes them through a sum, a difference, a product, a quotient and a formula, so you can say what a calculation built on rounded data is *guaranteed* to give.

By the end of this unit you can

• **Write down** the lower and upper bounds of a value rounded to a unit, to decimal places or to significant figures, and **explain** why the interval is half-open

• **Represent** those bounds as an inequality and on a number line

• **Calculate** the bounds of a sum and a difference, using the cross rule

• **Calculate** the bounds of a product and a quotient, using the flip — the divisor takes the bound at the opposite end

• **Calculate** the bounds of a quantity given by a formula with several rounded inputs

• **Justify** a yes-or-no decision from the worst case rather than the nominal case

The one idea behind all of it

Every rounded figure you are handed is really a pair of numbers. “15 m to the nearest metre” is not a length, it is the promise $14.5 \leq d < 15.5$, and the whole of this unit is one habit: carry BOTH walls of that promise through the calculation, decide at every step which wall helps and which hurts, and report the answer as an interval — because that is all the data ever supported.

LB = value − ½ unit and UB = value + ½ unit, and the interval is LB ≤ x < UB — closed at the bottom, open at the top. Four facts do almost all the work here: a rounded value is trapped between two exact bounds half a unit either side; adding and multiplying pair like with like while subtracting and dividing cross over; bounds are exact and are never rounded; and a guarantee compares the upper bound of what must fit with the lower bound of what it fits into.

Give every derived answer to 3 significant figures, unless it is exact — and give every amount of money in pula to 2 decimal places. Round, never truncate. A bound that is exact, such as 14.5 or 3 550 000, is written in full and used in full.

Where this unit sits

This unit stands on N04 Round It Right and N17 Accuracy on Trial. N04 taught you to round to a place value, to decimal places and to significant figures. N17 showed you informally that a rounded figure stands for an interval one rounding unit wide, and that the error travels through a calculation. Both are assumed here and neither is re-taught.

What is new is that the interval becomes the object you calculate with. N17 asked “how far out could this be?” and answered with a percentage. N18 asks “what is the largest the answer could possibly be, and the smallest?” and answers with two exact numbers — the upper bound and the lower bound — which is what an Extended paper asks for and what a tolerance decision actually needs.

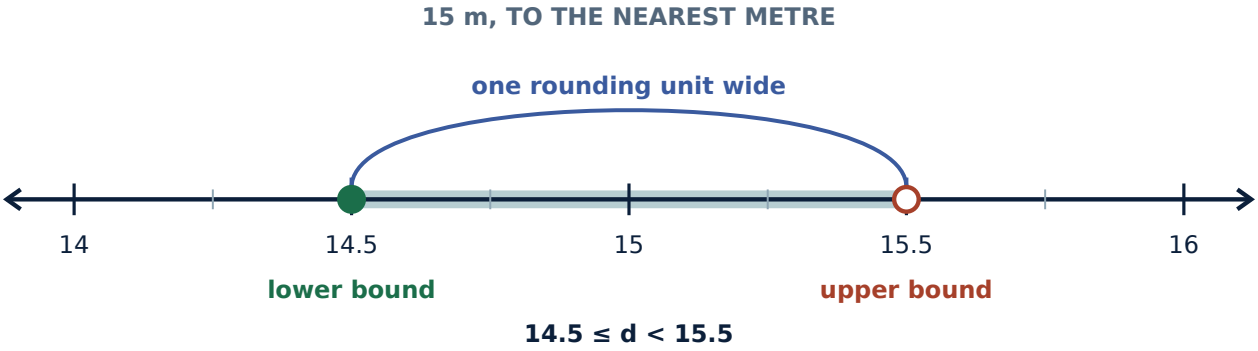
It also stands on N08 The Number Line and N14 Between the Lines. LB ≤ x < UB is a double inequality with a closed circle at one end and an open circle at the other, exactly as those units drew it. If the open and closed circles are shaky, go back to N08 first.

Concept 1 · The two bounds, and why the interval is half-open

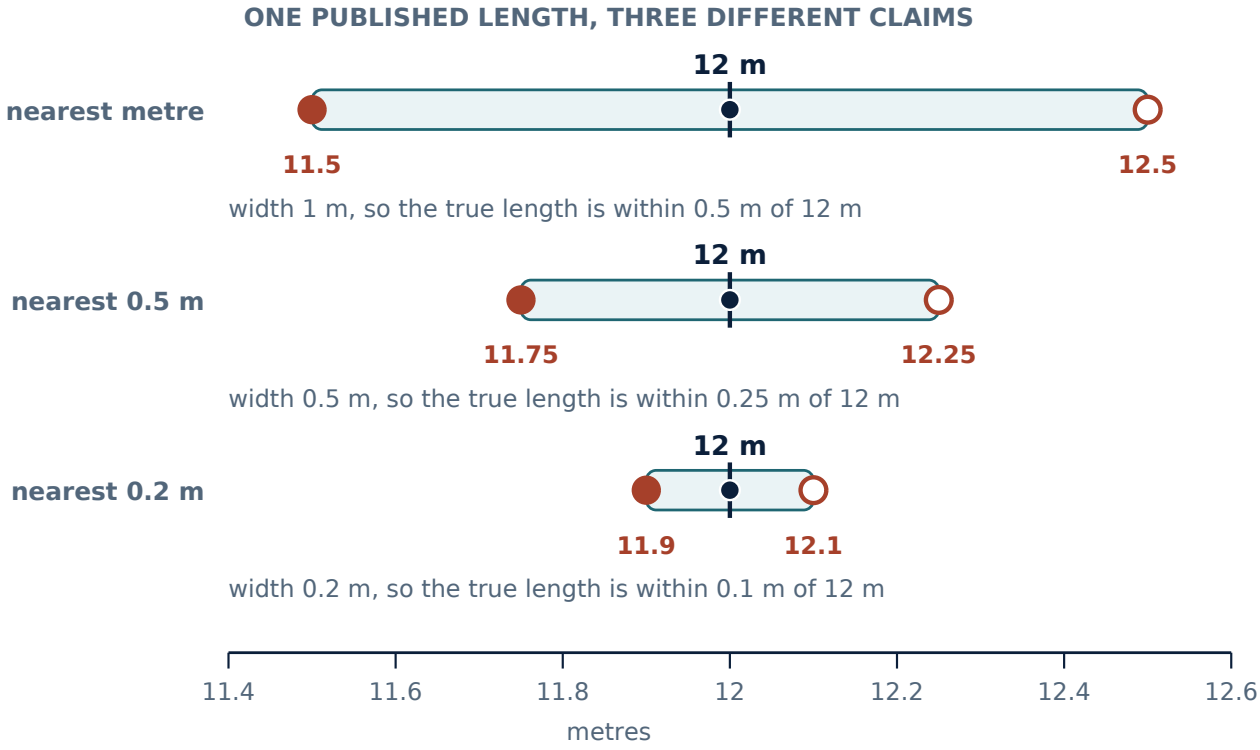
A rounded value is trapped between a lower bound it can reach and an upper bound it cannot

When a figure has been rounded, the truth is trapped in an interval half a rounding unit either side. The bottom is the lower bound and is included;

the top is the upper bound and is excluded, because it would round to the next figure up. That is why the interval is written LB ≤ x < UB and is called half-open.



The filled circle is included and the hollow circle is not, which is the whole meaning of "half-open". 14.5 m rounds to 15 m, so it is allowed; 15.5 m rounds to 16 m, so it is not.



The same 12 m, rounded three ways. Each bar is everything the figure allows the true length to be. The filled circle at the left is the lower bound, which the length can actually reach; the hollow circle at the right is the upper bound, which it cannot. The bar is exactly as wide as the unit it was rounded to.

Worked example 1 · What “15 m, to the nearest metre” traps the length between

1	Name the rounding unit that was used	**1 m**
2	Half the unit is the most the figure can be out by	**0.5 m**
3	The lower bound: half a unit below	**15 − 0.5 = 14.5 m**
4	The upper bound: half a unit above	**15 + 0.5 = 15.5 m**
5	Write it as one inequality	**14.5 ≤ d < 15.5**
6	Why the second sign is strict, not ≤	**15.5 rounds to 16, not to 15**

Two bounds and two different signs. The lower bound is a value the length can actually take: 14.5 m rounds to 15 m, so it is allowed, and that is why it gets ≤. The upper bound is a value the length can never take: 14.5 rounds up but so does 15.5, straight past 15 to 16, so 15.5 is excluded and gets <. The interval is **half-open**, it is written [14.5, 15.5), and it is exactly one rounding unit wide. Write both bounds down before you calculate anything, every single time.

What a rounded figure traps

As published	Rounded to	Unit	The interval	Width
15 m	nearest metre	1 m	14.5 ≤ d < 15.5	1 m
4.7 kg	nearest 0.1 kg	0.1 kg	4.65 ≤ m < 4.75	0.1 kg
12 m	nearest 0.5 m	0.5 m	11.75 ≤ r < 12.25	0.5 m
2 500 people	nearest 100	100	2 450 ≤ n < 2 550	100
7.30 s	2 decimal places	0.01 s	7.295 ≤ t < 7.305	0.01 s
3 600 000 m²	2 significant figures	100 000 m²	3 550 000 ≤ A < 3 650 000	100 000 m²

Concept 2 · Bounds from decimal places and significant figures

Find the rounding unit first; everything else is half of it

Decimal places and significant figures both do one job: they tell you the rounding unit. To n decimal places the unit is 10 to the power −n. To n significant figures, find the last significant digit and ask what it is worth. Then the bounds are half the unit, each way.

FIVE PUBLISHED FIGURES AND WHAT THEY TRAP

	rounded to	lower bound	upper bound
15 m	nearest metre	14.5 m	15.5 m
4.7 kg	nearest 0.1 kg	4.65 kg	4.75 kg
2 500	nearest 100	2 450	2 550
3 600 000 m²	2 s.f.	3 550 000 m²	3 650 000 m²
0.0250 kg	3 s.f.	0.02495 kg	0.02505 kg

find the rounding unit first, then halve it

Decimal places and significant figures are not two rules. Both of them just tell you the rounding unit, and the bounds are half of it, each way.

Worked example 2 · Bounds when the figure is given to significant figures: 3 600 000 m² to 2 s.f.

1	Find the last SIGNIFICANT digit	**the 6, standing for 6 hundred thousand**
2	That digit fixes the rounding unit	**100 000 m²**
3	Half the unit	**50 000 m²**
4	Lower bound	**3 600 000 − 50 000 = 3 550 000 m²**
5	Upper bound	**3 600 000 + 50 000 = 3 650 000 m²**
6	As a percentage, the most it can be out by	**50 000 ÷ 3 600 000 × 100 = 1.39%**

Significant figures are not a different rule, they are the same rule with one extra step: **find the rounding unit first**. The last significant digit tells you what the figure was rounded to, and everything after that is “half of it, each way”. Here 2 significant figures on a seven-digit number means the unit is 100 000, so an area printed as a single tidy number is really a claim about a hundred-thousand-square-metre band. The same method reads 0.0250 kg to 3 s.f. as 0.02495 to 0.02505 kg — leading zeros are never significant, so they never change the unit.

Concept 3 · Sums and differences: the cross rule

Add like for like; subtract crossways, because taking less away leaves more

For a sum you pair like with like. For a difference the second input crosses over, because taking away less leaves more: max = UB(a) − LB(b) and min = LB(a) − UB(b). A difference is exactly as wide as the corresponding sum — uncertainty never cancels.

WHICH BOUND OF EACH INPUT FEEDS WHICH BOUND

	lower bound of the answer	upper bound of the answer
a + b	LB(a) + LB(b)	UB(a) + UB(b)
a − b	LB(a) − UB(b)	UB(a) − LB(b)
a × b	LB(a) × LB(b)	UB(a) × UB(b)
a ÷ b	LB(a) ÷ UB(b)	UB(a) ÷ LB(b)

for − and ÷ the second input crosses over

Two operations keep like with like, and two cross over. Adding and multiplying pair the lower bounds together and the upper bounds together. Subtracting and dividing swap the second input to the opposite end, because taking away less leaves more and dividing by less gives more.

Worked example 3 · a − b when both are rounded: why the bounds cross

1	The two intervals, written out first	**8.55 ≤ a < 8.65 and 3.15 ≤ b < 3.25**
2	For a SUM you add like for like	**8.55 + 3.15 = 11.7 and 8.65 + 3.25 = 11.9**
3	For a DIFFERENCE, the largest answer takes away the least	**8.65 − 3.15 = 5.5**
4	and the smallest answer takes away the most	**8.55 − 3.25 = 5.3**
5	So the interval is	**5.3 ≤ a − b < 5.5**
6	Its width, like the sum's, is the two units added	**0.1 + 0.1 = 0.2**

The cross rule. For a + b you add like for like; for a − b you swap the second input over: max = UB(a) − LB(b) and min = LB(a) − UB(b). The reason is one sentence long: taking less away leaves more. And notice what does *not* happen — the uncertainties do not cancel. A difference is exactly as uncertain as the corresponding sum, 0.2 m wide either way. A student who subtracts like for like gets 5.4 for both bounds, an interval of zero width, which would mean the difference of two inexact measurements was known exactly.

Which bound feeds which

Operation	Lower bound	Upper bound	Why	For a = 8.6, b = 3.2
a + b	LB(a) + LB(b)	UB(a) + UB(b)	like with like	11.7 to 11.9
a − b	LB(a) − UB(b)	UB(a) − LB(b)	crossed — taking less away leaves more	5.3 to 5.5
a × b	LB(a) × LB(b)	UB(a) × UB(b)	like with like (for positive quantities)	26.9325 to 28.1125
a ÷ b	LB(a) ÷ UB(b)	UB(a) ÷ LB(b)	flipped — dividing by less gives more	2.63 to 2.75

Concept 4 · Products and quotients: the flip

The divisor takes the bound at the opposite end — the single most-failed step in the topic

A product of positive quantities pairs like with like. A quotient does not: dividing by a smaller number gives a bigger answer, so the upper bound of a ÷ b is UB(a) ÷ LB(b). This is the flip, and it is the single most-failed step in the topic.

ALL FOUR CORNERS OF a ÷ b, FOR a = 8.6 AND b = 3.2

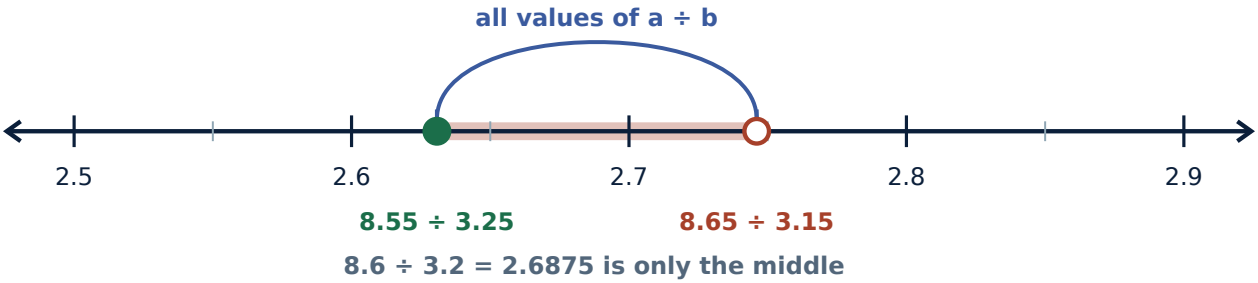
	UB ÷ UB	UB ÷ LB	LB ÷ UB	LB ÷ LB
the sum	8.65 ÷ 3.25	8.65 ÷ 3.15	8.55 ÷ 3.25	8.55 ÷ 3.15
its value, 3 s.f.	2.66	2.75	2.63	2.71
verdict	inside	GREATEST	LEAST	inside

the upper bound divides by the SMALLER divisor

both given to 1 decimal place

Every one of the four values is a value a ÷ b really could take. Only two of them are bounds, and the two in the middle are the ones a careless answer reaches for. Test all four corners whenever you are not sure.

a ÷ b, FOR a = 8.6 AND b = 3.2 TO 1 d.p.



8.6 ÷ 3.2 = 2.6875 is only the middle

The published quotient sits inside a band about 0.115 wide. Neither end of that band comes from dividing an upper bound by an upper bound.

Worked example 4 · a ÷ b when both are rounded: the flip

1	Try all four corner combinations	**8.65÷3.15, 8.65÷3.25, 8.55÷3.15, 8.55÷3.25**
2	The greatest: biggest top, SMALLEST bottom	**8.65 ÷ 3.15 = 2.746… = 2.75**
3	The least: smallest top, LARGEST bottom	**8.55 ÷ 3.25 = 2.630… = 2.63**
4	The tempting wrong answer, upper over upper	**8.65 ÷ 3.25 = 2.66 — inside, not the bound**
5	So the interval is	**2.63 ≤ a ÷ b < 2.75 (3 s.f.)**
6	A product does NOT flip: both bounds stay on the same side	**8.55×3.15 = 26.9325 and 8.65×3.25 = 28.1125**

The flip. Dividing by a smaller number gives a bigger answer, so the upper bound of a quotient is UB(a) ÷ LB(b) and the lower bound is LB(a) ÷ UB(b). This is the single most-failed step in the topic: the instinct is “upper bound means upper bounds everywhere”, and for a sum and a product that instinct is right, while for a difference and a quotient it is wrong. Two operations keep like with like (+ and ×) and two operations cross over (− and ÷). If you cannot remember which, test the four corners: it takes twenty seconds and it is never wrong.

How wide the answer comes out

Operation	How wide the answer is	For a = 8.6, b = 3.2
a + b	the two input widths ADDED	0.1 + 0.1 = 0.2
a − b	the two input widths ADDED, exactly as for a sum	0.1 + 0.1 = 0.2
a × b	grows with the SIZE of a and b	28.1125 − 26.9325 = 1.18
a ÷ b	grows with the size of a, and blows up as b nears zero	2.746... − 2.630... ≈ 0.115

Concept 5 · Bounds through a formula

Several rounded inputs, one answer, and a decision about each input in turn

A formula with several rounded inputs is the same problem repeated: go through the inputs one at a time and decide which bound helps. Multiplied on the top, take the lower bound for a lower bound; on the bottom, take the upper. An exact constant contributes nothing.

$E = Ad^2 \div 700$, WORKED AT BOTH ENDS

	A (m^2)	d (m)	E
lower bound	**3 550 000**	**14.5**	**1 070 000**
as published	**3 600 000**	**15**	**1 160 000**
upper bound	**3 650 000**	**15.5**	**1 250 000**

each input at its own lower bound

the 700 is exact and adds no uncertainty

Both inputs sit on the top of the fraction, so both of them push E the same way. The lower bound of E therefore takes the lower bound of each, and the upper bound takes the upper bound of each. Every figure in the E column is rounded once, at the end, to 3 significant figures.

Worked example 5 · The lower bound of E = Ad² ÷ 700, from two rounded inputs

1	Write both inputs as intervals first	**3 550 000 ≤ A < 3 650 000; 14.5 ≤ d < 15.5**
2	Decide what each input does to E	**both are on the top, so both increase E**
3	So the LOWER bound of E takes the lower bound of each	**A = 3 550 000 and d = 14.5**
4	Square the depth first, exactly	**$14.5^2 = 210.25$**
5	Multiply and divide by the exact constant	**$3\,550\,000 \times 210.25 \div 700 = 1\,066\,267.857\ldots$**
6	Round the ANSWER once, at the end	**lower bound of E = 1 070 000 (3 s.f.)**

A formula with several rounded inputs is not harder, it is just longer: **decide, input by input, which bound helps**. Anything multiplied on the top takes its lower bound for a lower bound; anything on the bottom takes its upper bound. The 700 is an exact constant and contributes nothing. Do the arithmetic on the exact bounds — 14.5 and 3 550 000, never rounded first — and round only the final answer. The same reasoning gives an upper bound of 1 250 000, so E is settled only to about 1.07 to 1.25 million: the published 1 160 000 has one honest significant figure in it.

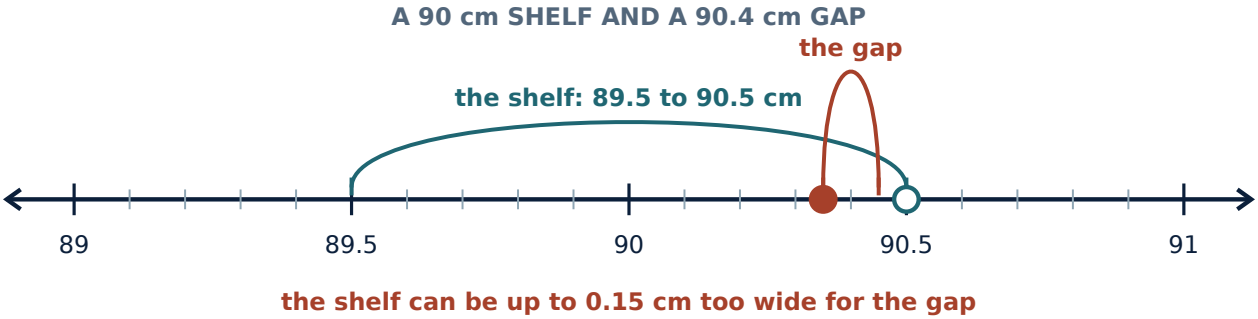
One formula, input by input

Input	As given	The interval	What it does to E
A, the area	3 600 000 m² to 2 s.f.	3 550 000 ≤ A < 3 650 000	on the top, so lower for lower
d, the depth	15 m to the nearest metre	14.5 ≤ d < 15.5	on the top and squared, so lower for lower
700	an exact constant	exactly 700	adds no uncertainty at all
E = Ad² ÷ 700	published as 1 160 000	1 070 000 to 1 250 000 (3 s.f.)	one honest significant figure

Concept 6 · Can you be certain? Working with the worst case

Compare the upper bound of what must fit with the lower bound of what it must fit into

A decision that has to be right cannot be taken on the published figures. Work with the worst case: compare the upper bound of what must fit with the lower bound of what it fits into. If UB(thing) ≤ LB(hole) you are certain; otherwise you are not.



The shelf's interval reaches past the gap's lower bound, so the two overlap and the fit is not guaranteed. Compare the upper bound of what must fit with the lower bound of what it fits into — every time.

Worked example 6 · “It is 90 cm and the gap is 90.4 cm” — can you be sure it fits?

1	The shelf, to the nearest centimetre	**89.5 ≤ s < 90.5**
2	The gap, to 1 decimal place	**90.35 ≤ g < 90.45**
3	The dangerous case: widest shelf, narrowest gap	**s up to 90.5 against g = 90.35**
4	Compare UB(shelf) with LB(gap)	**90.5 > 90.35**
5	So the answer is	**no — it is not certain to fit**
6	By how much it could be too wide	**up to 90.5 − 90.35 = 0.15 cm**

The nominal figures say 90 cm into 90.4 cm, a comfortable 0.4 cm of room. The bounds say the shelf could be almost 90.5 cm and the gap could be 90.35 cm, so it could be 0.15 cm too wide. **A guarantee is always the same comparison: the upper bound of what has to fit against the lower bound of what it fits into.** Everything else — the published figures, the averages, the “it should be fine” — is about the typical case, and nobody gets to choose the typical case. If UB(thing) ≤ LB(hole) you are certain; otherwise you are not, and the honest answer is “measure again”.

Four kinds of guarantee

The decision	What to compare	The test	Worked case
Will it fit into the gap?	UB(thing) against LB(gap)	certain if UB(thing) ≤ LB(gap)	the shelf: 90.5 against 90.35 — NOT certain
Is the load under the limit?	UB(load) against LB(limit)	certain if UB(load) ≤ LB(limit)	596 against 595 — NOT certain
Will it last long enough?	LB(supply) ÷ UB(rate)	round the answer DOWN	4 950 ÷ 47.5 = 104.2, so 104 days
How many will certainly fit?	n × UB(each) against LB(limit)	round the answer DOWN	595 ÷ 74.5 = 7.98, so 7 crates

The bounds routine

1. Write both bounds down before you calculate anything — Find the rounding unit, halve it, and write LB ≤ x < UB for every rounded input. A question you cannot start is nearly always a question whose intervals have not been written out.

2. Decide, input by input, which bound helps — For a bigger answer: bigger on the top, SMALLER on the bottom, and smaller for anything being subtracted. Like with like for + and ×; crossed over for − and ÷. If you are unsure, work out all four corners and take the biggest and the smallest.

3. Keep the bounds exact all the way through — 14.5 and 3 550 000 are exact consequences of what was written down, not estimates. Never round a bound, and never round a step. Exact constants such as the 700 in $Ad^2 \div 700$ add no uncertainty at all.

✓ **Round the answer once, and then answer the question** — Give a derived bound to 3 significant figures, money in pula to 2 decimal places. Then say what it means: for a guarantee, compare the upper bound of what must fit with the lower bound of what it fits into, and round a “how many” answer DOWN.

The mistakes that cost the most marks

Trap 1 · Using the upper bound of every input to get the upper bound of a quotient

WRONG “the biggest a ÷ b must be 8.65 ÷ 3.25 = 2.66”

RIGHT the divisor takes its LOWER bound: 8.65 ÷ 3.15 = 2.75 to 3 significant figures

This is the most expensive single error in the whole Extended bounds row, and it comes from a reasonable-sounding rule that happens to be wrong: “upper bound means upper bounds everywhere”. It is right for a sum and right for a product, and it is wrong for a difference and wrong for a quotient. The reason is not a convention to memorise, it is arithmetic you already trust: dividing a fixed amount among fewer people gives each person more. So the largest quotient puts the largest possible numerator over the smallest possible denominator, and the smallest quotient does the opposite. Here 8.65 ÷ 3.25 = 2.66 is a perfectly real value of a ÷ b — it is just not the largest one, and quoting it as the upper bound understates the answer by about 0.085, which in a tolerance decision is the difference between certain and not certain. When in doubt, work out all four corners and take the biggest and the smallest; it costs twenty seconds and it cannot go wrong.

Trap 2 · Subtracting like for like when finding the bounds of a difference

WRONG “lower bound = 8.55 − 3.15 and upper bound = 8.65 − 3.25, so a − b = 5.4 both ways”

RIGHT the bounds cross: min = 8.55 − 3.25 = 5.3 and max = 8.65 − 3.15 = 5.5

The giveaway that this method is broken is built into its own answer: both “bounds” come out as 5.4, so it claims that a difference of two measurements, neither of which is known exactly, is itself known exactly. That cannot be true, and the interval it produces has zero width. What goes wrong is the direction of the second input. In a + b, making b bigger makes the answer bigger, so like goes with like. In a − b, making b bigger makes the answer SMALLER, so the largest answer needs the smallest b. Written out: max = UB(a) − LB(b) and min = LB(a) − UB(b). Two further facts are worth carrying away. First, the width of a difference is the two input widths ADDED, exactly as for a sum — uncertainty never cancels. Second, a difference of two nearly equal rounded numbers is the most dangerous calculation in the topic, because the answer shrinks while its interval does not.

Trap 3 · Rounding the bounds themselves

WRONG “the lower bound of 3 600 000 m² to 2 s.f. is about 3.6 million, and 14.5 is about 15”

RIGHT bounds are EXACT: 3 550 000 and 14.5 go into the calculation untouched, and only the final answer is rounded

A bound is not a measurement and it is not an estimate. It is an exact consequence of what somebody wrote down: if an area is given to 2 significant figures as 3 600 000 m², then its lower bound is exactly 3 550 000 m², not approximately, and 14.5 m is exactly the lower bound of a depth given as 15 m to the nearest metre. Rounding a bound does two kinds of damage. It destroys the guarantee — a lower bound that has been rounded up is no longer a lower bound, and a decision built on it is no longer safe. And it corrupts the arithmetic before it starts, so the error grows through every step that follows. There is exactly one place in a bounds question where rounding is allowed: the final derived answer, once, at the end, to 3 significant figures. The width of an interval is also computed from the exact bounds — here 1 252 732.142… − 1 066 267.857… = 186 464.285…, so 186 000 to 3 s.f., and never 1 250 000 − 1 070 000.

Practice · Fluency · 24 questions

P1 A length is given as 15 m, to the nearest metre. **Write down** the lower bound.

P2 **Write down** the upper bound of that same length.

P3 **Write down** that length as an inequality, using L for the true length.

P4 **State** why the upper bound itself is not a possible value of L.

P11 An area is given as 3 600 000 m², to 2 significant figures. **Write down** the rounding unit that implies.

P12 **Write down** the lower bound of that area.

P13 **Write down** the upper bound of that area.

P14 **Write down** the bounds of a mass given as 0.0250 kg, to 3 significant figures.

P21 Two lengths are a = 8.6 m and b = 3.2 m, each given to 1 decimal place. **Write down** the upper bound of a + b.

P22 **Write down** the lower bound of a + b.

P23 **Write down** the upper bound of a − b.

P24 **Write down** the lower bound of a − b.

P31 **Write down** the upper bound of a × b, where a = 8.6 m and b = 3.2 m, each given to 1 decimal place.

P32 **Write down** the lower bound of a × b.

P33 **State** which bound of b gives the GREATEST value of a ÷ b.

P34 **Write down** the calculation that gives the upper bound of a ÷ b.

P41 In E = Ad² ÷ 700, the area A is 3 600 000 m² to 2 significant figures. **Write down** the lower bound of A.

P42 The depth d is 15 m, to the nearest metre. **Write down** the lower bound of d.

P43 **State** which bound of A and which bound of d are needed for the lower bound of E.

P44 **Calculate** E from the published figures, to 3 significant figures.

P51 A shelf is 90 cm long, to the nearest centimetre. **Write down** its upper bound.

P52 The gap it must fit into is 90.4 cm, to 1 decimal place. **Write down** the lower bound of the gap.

P53 **State** whether the shelf is certain to fit.

P54 **State** which bound of the shelf and which bound of the gap must be compared to be certain.

Practice · Applied · 24 questions

P5 A mass is recorded as 4.7 kg, to the nearest 0.1 kg. **Determine** its bounds and the width of the interval.

P6 A crowd is reported as 2 500 people, to the nearest 100. **Determine** the bounds.

P7 A rope is measured as 12 m, to the nearest 0.5 m. **Determine** its bounds.

P8 A distance is given as 8.0 km, to 1 decimal place. **Determine** its bounds, and **explain** what the zero is doing there.

P15 A time is recorded as 7.30 s, to 2 decimal places. **Determine** its bounds.

P16 A population is given as 48 000, to 2 significant figures. **Determine** the bounds and the width of the interval.

P17 One speed is quoted as 25 m/s to 2 significant figures and another as 25.0 m/s to 3 significant figures. **Determine** both intervals and **state** which is the stronger claim.

P18 **Calculate** the greatest possible error in 3 600 000 m² as a percentage of the published area, to 3 significant figures.

P25 **Determine** the interval for a + b and its width.

P26 **Determine** the interval for a − b and its width, and **explain** the crossing.

P27 Two planks, 2.4 m and 1.8 m each to the nearest 0.1 m, are laid end to end. **Determine** the bounds of the total length.

P28 A tank holds 250 litres, to the nearest 10 litres. 85 litres, to the nearest 5 litres, is drawn off. **Determine** the bounds of what is left.

P35 **Determine** the upper bound of a ÷ b, to 3 significant figures.

P36 **Determine** the lower bound of a ÷ b, to 3 significant figures.

P37 A car covers 250 km, to the nearest 10 km, in 3.5 hours, to the nearest 0.1 hour. **Determine** the bounds of its average speed, to 3 significant figures.

P38 A rectangle measures 12.4 cm by 7.8 cm, each to 1 decimal place. **Determine** the bounds of its area exactly.

P45 **Calculate** the lower bound of E, to 3 significant figures.

P46 **Calculate** the upper bound of E, to 3 significant figures.

P47 **Determine** the width of the interval for E, to 3 significant figures.

P48 A box measures 2.4 m by 1.5 m by 0.8 m, each to 1 decimal place. **Determine** the bounds of its volume, to 3 significant figures.

P55 **Determine** by how much the shelf could be too wide.

P56 A box is 34 cm wide, to the nearest centimetre. A doorway is 35 cm wide, to the nearest centimetre. **Determine** whether the box is certain to pass through, and **justify** your answer.

P57 A hoist is rated at 600 kg, to the nearest 10 kg. Eight crates each weigh 72 kg, to the nearest kilogram. **Determine** whether the load is certainly safe.

P58 The same hoist is loaded with eight crates each weighing 74 kg, to the nearest kilogram. **Determine** whether the load is certainly safe, and **comment** on what the nominal figures suggest.

Practice · Challenge · 12 questions

P9 The lower bound of a length rounded to the nearest 0.2 m is 6.3 m. **Determine** the length as it was published, and its upper bound.

P10 A value x satisfies $24.5 \le x < 25.5$. **Determine** what was published and to what unit, and **determine** how many values written to 1 decimal place lie in the interval.

P19 A number rounded to 3 significant figures is 0.0504. **Determine** its bounds, and **explain** why the leading zeros make no difference.

P20 A value v satisfies $0.4835 \le v < 0.4845$. **Determine** what was published, and to how many significant figures.

P29 Three lengths, each 6.0 m to 1 decimal place, are joined. **Determine** the bounds of the total and **state** the width.

P30 A student finds the bounds of $a - b$ by subtracting like for like: $8.55 - 3.15$ and $8.65 - 3.25$. **Determine** what they get, and **explain** why the answer is nonsense.

P39 Density is mass ÷ volume. A block has mass 48 g, to the nearest gram, and volume 15 cm^3, to the nearest cm^3. **Determine** the bounds of its density, to 3 significant figures.

P40 **Explain** why the upper bound of $a \div b$ uses the LOWER bound of b, and **determine** what a student gets by dividing upper bound by upper bound.

P49 **Determine** the greatest possible percentage error in the published value of E, to 3 significant figures.

P50 **Determine** which of the two inputs E is more sensitive to, and **justify** your answer with figures.

P59 **Determine** the greatest whole number of those 74 kg crates that the hoist is certain to carry.

P60 A tank holds 5 000 litres, to the nearest 100 litres. A household draws 45 litres a day, to the nearest 5 litres. **Determine** the greatest number of whole days the tank is certain to last.

Criterion B · Investigating patterns

How wide is the answer?

Two quantities are measured and rounded, and then something is done with them. The answer is no longer a number either — it is an interval, and that interval has a **width**. **How does the width of the answer depend on what you did to the two quantities?** Adding, subtracting, multiplying and dividing all take the same two intervals in and all give an interval out, so it is a fair race. Two of the four behave identically and the other two do not behave like each other at all. The only tools you need are the bound rules from this unit and a calculator.

THE TABLE YOU HAVE TO COMPLETE

	a + b	a − b	a × b	a ÷ b
lower bound	16			
upper bound	18			
width	2			

a = 12 and b = 5, each to the nearest whole number

One column is given. Work the other three out yourself, keeping every value exact, and then look hard at the bottom row.

Part 1 · structured

Operation	Lower bound	Upper bound	Width
a + b	16	18	2
a − b			
a × b			
a ÷ b			

1. **Predict**, before you calculate anything, which of the four operations will give the WIDEST interval and which the narrowest, and say whether you expect a + b and a − b to come out the same or different. Write the prediction down; you will be asked to compare it with what you find.

2. **Complete** the table for a = 12 and b = 5, each given to the nearest whole number, so that each is trapped between bounds one unit apart.

For each of a + b, a − b, a × b and a ÷ b give the lower bound, the upper bound and the width. Keep the sums and differences exact; give the quotient to 3 significant figures but work out its width from the EXACT bounds.

3. **Identify** the pattern. What do the widths of a + b and a − b have in common, and what do they have in common with the two rounding units you started from? Then look at the product and the quotient: say, with numbers, how their widths compare with the sizes of a and b themselves.

4. **Determine** a general rule for the width of the answer when two quantities with interval widths w(a) and w(b) are added, and another for when they are subtracted. Write each rule as a formula in w(a) and w(b), not in words alone. Then **determine** a rule for the width of a × b as well, in terms of a, b, w(a) and w(b).

5. **Test and verify** your rules on a pair that is not in the table: a = 7.4 and b = 2.5, each given to 1 decimal place. Then test them again on a = 12 and b = 5 given to the nearest 0.5. Say how close each rule came, and whether the sum rule and the difference rule still agree with each other. A rule tested only on the numbers it was built from has not been tested.

6. **Justify** your rules. Explain *why* a difference must widen by exactly as much as a sum, when subtracting looks as though it ought to cancel some of the uncertainty out. Then explain why the product's width depends on the SIZE of the two quantities while the sum's does not, and say what that means for a calculation that multiplies two large rounded numbers together.

Part 2 · open

Now you choose what to investigate. Take one of these openings or propose your own. You decide which quantities to try, how many you need, how to record them, and how you will test whatever you find.

1. Chain a third quantity on. **Investigate** what happens to the width when three rounded quantities are added, and when three are multiplied. **Determine** a rule for n quantities added, **test** it, and say whether the same kind of rule can work for n quantities multiplied.

2. Squeeze the divisor. **Investigate** the width of a ÷ b as b gets smaller while its rounding unit stays the same — try b = 5, 2, 1 and 0.5, each to the nearest 0.1. **Determine** what happens to the width as the divisor's lower bound approaches zero, **justify** what you find, and state the value

of b at which you would refuse to quote an answer at all.

3. Break the difference. **Investigate** what happens when a and b are nearly equal — for example 8.6 and 8.4, each to 1 decimal place. Compare the width of $a - b$ with the answer itself, **determine** how the two are related, and **justify** why a difference of two nearly equal rounded numbers is the most dangerous calculation in this unit.

Whichever you choose, take it through the same ladder: **predict**, **identify** the pattern, **determine** a general rule, **test and verify** it on cases you have not already used, and **justify** why it must hold and where it stops holding.

Marked out of 8 against Criterion B. Completing the table earns the lower bands; the general rules, the verification on the 7.4 and 2.5 pair and on the nearest-0.5 pair, and the justification are what reach levels 5–8. Part 2 must show your own choice of method.

Remember, you should communicate in an organized and coherent manner.

Criterion D · Applying maths in real-life contexts

Will it fit, and will it be legal?

A builders' yard in Gaborone has a delivery to send out this morning: a moulded water tank, eight bags of cement and a bundle of roofing sheets, on one flat-bed trailer, to a customer about 90 km away. The yard manager has a data sheet, and **the data sheet tells you far more than you need**. Two questions have to be answered before the trailer leaves, and both of them are yes-or-no: **will the tank fit across the bed**, and **is the load certainly inside the trailer's rating**? Every figure on the sheet is a rounded measurement, so neither question can be answered from the nominal numbers alone.

The data sheet (illustrative figures, written for this task)

• **Tank base diameter:** 1.9 m, to the nearest 0.1 m

• **Tank height:** 2.3 m, to the nearest 0.1 m

• **Tank stated capacity:** 5 000 litres, to the nearest 100 litres

• **Empty tank mass:** 78 kg, to the nearest kilogram

• **Trailer bed:** 2.0 m wide and 3.6 m long, each to the nearest 0.1 m

• **Trailer load limit:** 750 kg, to the nearest 50 kg

• **Also on the trailer:** 8 bags of cement at 50 kg each, to the nearest kilogram, and roofing sheets totalling 240 kg, to the nearest 10 kg

• **Yard gateway:** 2.4 m wide, to the nearest 0.1 m

• **Delivery fee:** P450.00, a fixed charge (exact)

• **Diesel:** P22.40 a litre on the day (exact)

• **Fuel use:** 14 litres per 100 km, to 2 significant figures

• **Distance to the customer:** about 90 km each way, to the nearest 10 km

• **Customer's roof catchment:** 96 m², to 2 significant figures

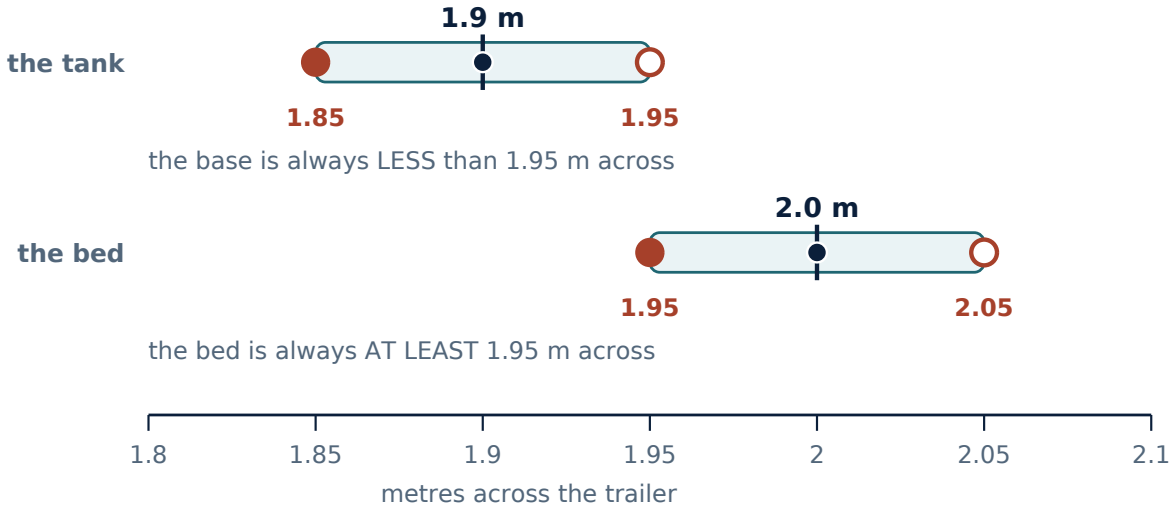
• **Last season's rainfall:** 380 mm, to 2 significant figures

• **Straps carried:** 6, rated at 400 kg each

• **Yard hours:** 07:00–17:00 on weekdays, 07:00–12:00 on Saturday

These figures are illustrative. The yard, the customer, the load, the prices and the distances were invented for this task. No real business, person, product or price list is named or implied, nothing here is a quotation, and there is no source to look up. What is authentic is the shape of the problem: tolerance decisions really are made on rounded measurements, and the nominal figures really do mislead.

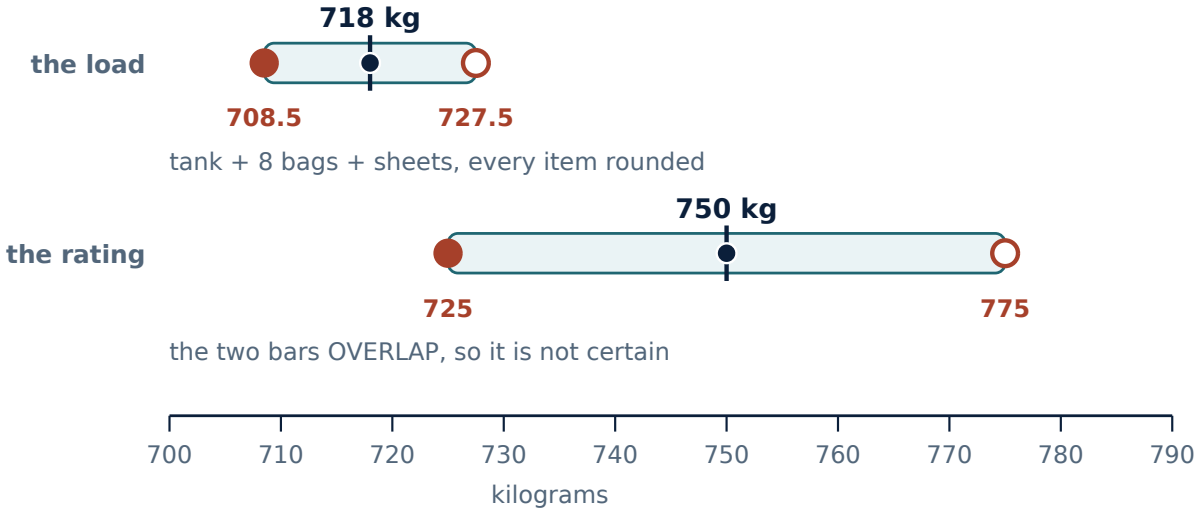
DOES THE TANK FIT ACROSS THE BED? THE TWO BARS MEET



illustrative figures, written for this task

The two intervals touch at 1.95 m and do not overlap. Because the tank's upper bound is open and the bed's lower bound is closed, the tank is always strictly narrower than the bed — certain to fit, and certain only because the interval is half-open.

718 kg AGAINST 750 kg LOOKS SAFE. THE BOUNDS DO NOT.



illustrative figures, written for this task

The nominal load is 718 kg and the nominal rating is 750 kg, so the published figures promise 32 kg of room. The load can reach 727.5 kg and the rating can be as low as 725 kg, so the bars overlap and the trip is not certain to be legal.

Identify which figures on the data sheet are rounded and to what, and write down the interval each one stands for. Then **justify**, item by item, which entries cannot affect either decision — deal with the yard gateway, the roof catchment, the rainfall and the yard hours by name — and **state** which two figures on the sheet are exact. *(Strand i — identifying the relevant elements)*

Select the strategies you will use and **justify** each choice. For the fit, say which bound of the tank and which bound of the bed decide it, and why. For the load, say which bound of every item in the load and which bound of the rating decide it, and why. **Explain** why the comparison is never between two nominal figures. *(Strand ii — selecting appropriate strategies)*

Apply them. **Calculate** (a) the intervals for the tank's base and the trailer bed, and decide the fit; (b) the interval for the total mass on the trailer, and compare it with the interval for the rating; (c) the greatest number of cement bags that is certainly safe; and (d) the cost of a second delivery run, in pula to 2 decimal places, as an interval — fuel plus the fixed fee. *(Strand iii — applying the strategy)*

Justify the accuracy of your two decisions. For each one, say which bound is the DANGEROUS bound and what would go wrong if you used the other. **Explain** exactly why the nominal figures mislead here: 718 kg against 750 kg looks like 32 kg of room, so **determine** what the worst case really is and how it can be worse than the published sum of the published parts. **Discuss** the fit decision, where the two bounds meet at 1.95 m: does that make the answer yes, no, or undecidable, and what does the half-open interval have to do with it? Finally, **justify** why none of the bounds may be rounded on the way through. *(Strand iv — accuracy of the solution)*

Justify a recommendation to the yard manager, who has to decide before the trailer leaves. State clearly what you would send today and what you would hold back, **justify** it with your intervals rather than with the nominal figures, and put a price on being wrong by comparing the cost of a second run with the consequence of an overloaded trailer. Then **discuss** the limitations of your own answer: which single extra measurement, taken more precisely, would change the decision most, and what have you assumed that the data sheet does not actually say? *(Strand v — does it make sense in context)*

Submit: your interval list for every relevant figure, your two decisions each with the comparison that settles it, the greatest safe number of bags, the cost interval for a second run, and written answers for strands i, iv and v. **Marked out of 8**

against Criterion D at Extended level, where the command term is **justify** rather than describe. Give money in pula to 2 decimal places and everything else to 3 significant figures, round rather than truncate, and never round a bound.

Answers

How the P-numbers run. The 60 practice items are numbered P1 to P60 in the order they appear in the lesson — ten per concept, and inside each concept four Fluency, then four Applied, then two Challenge. The same P-number identifies the item in the lesson, in this key and in the teacher pack, so the numbers inside each tier below step in blocks of ten rather than running consecutively: Fluency is P1–P4, P11–P14, P21–P24 and so on; Applied is P5–P8, P15–P18, …; Challenge is P9–P10, P19–P20, … Bounds are given exactly; derived answers are given to 3 significant figures unless they are exact.

Fluency · F-tier (24)

P1	**14.5 m**
P2	**15.5 m**
P3	**$14.5 \leq L < 15.5$**
P4	**because 15.5 would round up to 16, not to 15**
P11	**100 000 m²**
P12	**3 550 000 m²**
P13	**3 650 000 m²**
P14	**0.02495 kg and 0.02505 kg**
P21	**11.9 m**
P22	**11.7 m**
P23	**5.5 m**
P24	**5.3 m**
P31	**28.1125 m²**
P32	**26.9325 m²**
P33	**the lower bound of b**
P34	**8.65 ÷ 3.15**
P41	**3 550 000 m²**

P42	**14.5 m**
P43	**the lower bound of each**
P44	**1 160 000**
P51	**90.5 cm**
P52	**90.35 cm**
P53	**no**
P54	**the upper bound of the shelf against the lower bound of the gap**

Applied — fully worked (24)

P5 4.65 kg ≤ m < 4.75 kg; width 0.1 kg

Half the rounding unit is 0.05 kg. Take it off 4.7 for the lower bound and add it on for the upper bound: 4.65 kg and 4.75 kg. The width is 4.75 − 4.65 = 0.1 kg, which is the rounding unit itself — always.

P6 2 450 ≤ n < 2 550

Half of 100 is 50, so the true number is at least 2 450 and less than 2 550. Note that 2 550 is excluded: it would round to 2 600. The interval is 100 people wide, so the report can be 50 people out either way.

P7 11.75 m ≤ r < 12.25 m

Half of 0.5 is 0.25. So r is at least 12 − 0.25 = 11.75 m and less than 12 + 0.25 = 12.25 m. A finer rounding unit buys a narrower claim: the same rope to the nearest metre would only be pinned between 11.5 m and 12.5 m.

P8 7.95 km ≤ d < 8.05 km

To 1 decimal place the rounding unit is 0.1 km, so half of it is 0.05 km and the bounds are 7.95 km and 8.05 km. The zero after the point is not decoration: “8.0 km” claims the distance to within 50 m, while “8 km” to the nearest kilometre only claims it to within 500 m.

P15 7.295 s ≤ t < 7.305 s

To 2 decimal places the rounding unit is 0.01 s, so half of it is 0.005 s. The bounds are 7.30 − 0.005 = 7.295 s and 7.30 + 0.005 = 7.305 s. The trailing zero is significant and is what makes this claim ten times tighter

than “7.3 s”.

P16 47 500 ≤ P < 48 500; width 1 000

With 2 significant figures the last significant digit of 48 000 is the 8, which stands for 8 thousands, so the rounding unit is 1 000. Half of it is 500, giving 47 500 and 48 500. The width is the rounding unit, 1 000 people.

P17 24.5 to 25.5, and 24.95 to 25.05; the second is ten times stronger

To 2 significant figures the unit is 1, so the bounds are 24.5 m/s and 25.5 m/s. To 3 significant figures the unit is 0.1, so the bounds are 24.95 m/s and 25.05 m/s. The second interval is a tenth as wide, so the extra written zero is a claim about the instrument, not a decoration.

P18 1.39%

The rounding unit is 100 000 m², so the greatest possible error is half of it: 50 000 m². As a percentage of the published figure, 50 000 ÷ 3 600 000 × 100 = 1.3888…%, which is 1.39% to 3 significant figures.

P25 11.7 m ≤ a + b < 11.9 m; width 0.2 m

a lies between 8.55 m and 8.65 m and b between 3.15 m and 3.25 m. For a sum you add like for like: smallest with smallest, 8.55 + 3.15 = 11.7 m, and largest with largest, 8.65 + 3.25 = 11.9 m. The width is 0.2 m, which is exactly the two input widths added: 0.1 + 0.1.

P26 5.3 m ≤ a − b < 5.5 m; width 0.2 m

For a difference the bounds cross. The largest answer comes from the largest a and the SMALLEST b: 8.65 − 3.15 = 5.5 m — taking less away leaves more. The smallest answer comes from the smallest a and the largest b: 8.55 − 3.25 = 5.3 m. The width is 0.2 m, the same as for the sum: subtracting does not cancel the uncertainty, it adds it.

P27 4.1 m ≤ T < 4.3 m

Each plank has bounds half a unit either side: 2.35 m to 2.45 m and 1.75 m to 1.85 m. The least total is 2.35 + 1.75 = 4.1 m and the greatest is 2.45 + 1.85 = 4.3 m. The published total, 4.2 m, is carrying two roundings, so its interval is 0.2 m wide rather than 0.1 m.

P28 157.5 litres ≤ V < 172.5 litres

The tank holds between 245 and 255 litres; the amount drawn off is between 82.5 and 87.5 litres. Cross over: the most left is 255 − 82.5 = 172.5 litres and the least left is 245 − 87.5 = 157.5 litres. The interval is 15 litres wide, which is 10 + 5 — the two rounding units added.

P35 2.75

Dividing by a smaller number gives a bigger answer, so the greatest quotient uses the largest a and the SMALLEST b: 8.65 ÷ 3.15 = 2.746031…, which is 2.75 to 3 significant figures. Using 8.65 ÷ 3.25 instead gives 2.66 — a real value of the quotient, but not its upper bound.

P36 2.63

The least quotient uses the smallest a and the LARGEST b: 8.55 ÷ 3.25 = 2.630769…, which is 2.63 to 3 significant figures. Notice that each bound of the answer uses one bound from the top and the opposite bound from the bottom: that is the flip.

P37 69.0 km/h to 73.9 km/h

The distance lies between 245 km and 255 km and the time between 3.45 h and 3.55 h. Greatest speed = 255 ÷ 3.45 = 73.9130… = 73.9 km/h: the longest distance in the shortest time. Least speed = 245 ÷ 3.55 = 69.0140… = 69.0 km/h. The published figure, 250 ÷ 3.5, is about 71.4 km/h, comfortably inside.

P38 95.7125 cm² ≤ A < 97.7325 cm²

The sides lie between 12.35 cm and 12.45 cm, and between 7.75 cm and 7.85 cm. For a product of two positive quantities the bounds do not cross: least × least = 12.35 × 7.75 = 95.7125 cm² and greatest × greatest = 12.45 × 7.85 = 97.7325 cm². Both bounds are exact and neither is rounded.

P45 1 070 000

Every input appears on the top of the fraction and every input increases E, so the lower bound of E needs the lower bound of each: A = 3 550 000 and d = 14.5. Then $14.5^2 = 210.25$ and 3 550 000 × 210.25 ÷ 700 = 1 066 267.857…, which is 1 070 000 to 3 significant figures. The bounds 3 550 000 and 14.5 are used exactly — they are never rounded first.

P46 1 250 000

Take the upper bound of each input: A = 3 650 000 and d = 15.5. Then $15.5^2 = 240.25$ and 3 650 000 × 240.25 ÷ 700 = 1 252 732.142…, which is 1 250 000 to 3 significant figures. The 700 is an exact constant, so it contributes no uncertainty at all.

P47 186 000

The width is the upper bound minus the lower bound: 1 252 732.142… − 1 066 267.857… = 186 464.285…, which is 186 000 to 3 significant figures. The subtraction is done on the exact bounds, not on the rounded ones —

using 1 250 000 − 1 070 000 = 180 000 would be rounding the bounds and then rounding again.

P48 2.56 m³ to 3.23 m³

Every edge lies half a unit either side: 2.35 to 2.45, 1.45 to 1.55, and 0.75 to 0.85. All three multiply, so least × least × least = 2.35 × 1.45 × 0.75 = 2.555625 m³ = 2.56 m³, and greatest gives 2.45 × 1.55 × 0.85 = 3.227875 m³ = 3.23 m³. Three roundings compound: the interval is a quarter of the answer wide.

P55 by up to 0.15 cm

The shelf can be almost 90.5 cm and the gap can be as narrow as 90.35 cm, so in the worst case the shelf is 90.5 − 90.35 = 0.15 cm too wide. It is “up to” 0.15 cm because 90.5 cm is the upper bound and is not itself attainable. The nominal figures, 90 cm into 90.4 cm, look like a comfortable 0.4 cm of room.

P56 yes, it is certain

The box is less than 34.5 cm and the doorway is at least 34.5 cm. The two bounds are equal, and that is enough: the box's upper bound is not attainable, so the box is always strictly narrower than 34.5 cm, while the doorway is always at least 34.5 cm. Every possible box fits every possible doorway — but only just, and only because the interval is half-open at the top.

P57 yes: at most 580 kg against a limit of at least 595 kg

Each crate is under 72.5 kg, so eight of them come to less than 8 × 72.5 = 580 kg. The rating is at least 600 − 5 = 595 kg. Since 580 ≤ 595, every possible load is under every possible limit, so the lift is certainly safe with 15 kg to spare in the worst case.

P58 no: up to 596 kg against a limit of as little as 595 kg

Each crate can be almost 74.5 kg, so the load can be almost 8 × 74.5 = 596 kg, while the rating can be as low as 595 kg. Since 596 > 595, there are possible loads that exceed possible limits, so it is not certain. The nominal arithmetic, 8 × 74 = 592 kg against 600 kg, suggests 8 kg of comfort that the measurements do not actually guarantee.

Challenge — fully worked (12)

P9 6.4 m; upper bound 6.5 m

Half the rounding unit is 0.1 m, and the lower bound sits half a unit below the published figure, so the published length is 6.3 + 0.1 = 6.4 m. The upper bound is half a unit above it: 6.4 + 0.1 = 6.5 m. Check: the interval runs 6.3 to 6.5, which is 0.2 m wide, as it must be.

P10 25, to the nearest whole number; 10 values

The interval is 1 wide, so the rounding unit is 1 and the published value is the midpoint, 25. The 1-decimal-place values inside are 24.5, 24.6, …, 25.4 — ten of them. 25.5 is not one of them, because the upper bound is excluded: it rounds to 26.

P19 0.05035 ≤ x < 0.05045

The three significant figures are 5, 0 and 4; the zeros in front of the 5 only place the number and are not significant. The last significant digit is the 4, which stands for 4 ten-thousandths, so the rounding unit is 0.0001 and half of it is 0.00005. The bounds are 0.05035 and 0.05045.

P20 0.484, to 3 significant figures

The interval is 0.001 wide, so the rounding unit is 0.001 and the published value is the midpoint, 0.484. For 0.484 the last significant digit is the 4, standing for 4 thousandths, so the rounding unit 0.001 means the figure was given to 3 significant figures.

P29 17.85 m ≤ T < 18.15 m; width 0.3 m

Each length lies between 5.95 m and 6.05 m. Three smallest give 3 × 5.95 = 17.85 m and three largest give 3 × 6.05 = 18.15 m. The width is 0.3 m: three roundings, three tenths. Every measurement you add to a total adds its own rounding unit to the width of the answer.

P30 both give 5.4, so they get an interval of zero width

8.55 − 3.15 = 5.4 and 8.65 − 3.25 = 5.4. The two “bounds” are the same number, so the method claims the difference is known exactly — from two measurements that are not known exactly. That is impossible, and it is the giveaway that the bounds must cross: the true interval is 5.3 to 5.5, and it is 0.2 m wide.

P39 3.06 g/cm³ to 3.34 g/cm³

The mass lies between 47.5 g and 48.5 g; the volume between 14.5 cm³ and 15.5 cm³. Greatest density = 48.5 ÷ 14.5 = 3.344827… = 3.34 g/cm³: heaviest possible mass squeezed into the smallest possible volume. Least density = 47.5 ÷ 15.5 = 3.064516… = 3.06 g/cm³. The published 48 ÷ 15 = 3.2 g/cm³ is only the middle of a wide interval.

P40 2.66, which is a possible value but not the upper bound

Dividing a fixed amount among fewer people gives each more; dividing by a smaller number gives a bigger answer. So the greatest quotient pairs the greatest numerator with the least denominator. A student who divides upper by upper gets 8.65 ÷ 3.25 = 2.661538... = 2.66, which is genuinely one of the values a ÷ b could take — it is just not the largest, and quoting it as the upper bound understates the answer by about 0.085.

P49 8.26%

The published E is 1 157 142.857…. Upwards, the upper bound is 1 252 732.142…, a gap of 95 589.285…, which is 8.2608…% of the published value. Downwards, the gap to the lower bound is 90 875, which is 7.8534…%. The greatest possible error is therefore 8.26% to 3 significant figures — and the interval is NOT symmetric about the published figure, because d is squared.

P50 d, the depth

Compare each input's half-width as a percentage of itself. For A: 50 000 ÷ 3 600 000 × 100 = 1.39%. For d: 0.5 ÷ 15 × 100 = 3.33%. So d is already the shakier measurement — and it is then squared, which roughly doubles its effect to about 6.67%. Measuring the depth to the nearest 0.1 m would shrink the interval for E far more than an extra significant figure on the area would.

P59 7 crates

The worst case is n crates of almost 74.5 kg against a limit of 595 kg, so we need n × 74.5 ≤ 595, that is n ≤ 7.986…. The greatest whole number is 7, and 7 × 74.5 = 521.5 kg, safely under 595 kg. Eight crates reach 596 kg, which is over. Notice that the answer is rounded DOWN, whatever the decimal: rounding 7.986 up to 8 would put people under a load that can exceed the rating.

P60 104 days

The worst case is the least water with the fastest use: at least 4 950 litres, drawn at up to 47.5 litres a day. 4 950 ÷ 47.5 = 104.2105… days, so 104 whole days are guaranteed. The published figures, 5 000 ÷ 45 = 111.1 days, promise a week more than the measurements can support — and rounding 104.2 up to 105 would be a promise the tank cannot keep.