ALGEBRA · UNIT 1

From Numbers to Letters

Algebra looks like a new language, but it is just arithmetic with one twist: a letter stands in for a number you do not know yet. Master that one twist and everything in this course opens up.

Foundations · MYP 1–3 · ~40 minutes · 60 practice questions · Criteria A · B · C · D

BY THE END OF THIS UNIT YOU CAN

- ✓ Turn word descriptions like “5 more than n” into algebra
- ✓ Use algebra shorthand: write 3 × n as 3n and n ÷ 4 as n/4
- ✓ Substitute a number into an expression and evaluate it
- ✓ Handle expressions with two different letters
- ✓ Substitute negative numbers without sign slips
- ✓ Use BIDMAS so $3n^2$ and $(3n)^2$ never get confused

THE ONE IDEA BEHIND ALL OF IT

A letter in algebra is a box with a number hiding inside. The expression tells you what will happen to that number, whatever it turns out to be:

if n = 4, then 3n + 2 = 3 × 4 + 2 = 14

Swap the letter for the number, then calculate with BIDMAS as usual.

That swap is called substitution. Every formula you will ever use — areas, speeds, phone tariffs, exam grades — works exactly this way.

1 Saying it in symbols

Turn everyday words into an expression

Read the words slowly and build the expression piece by piece. “5 more than n” means start at n, add 5: n + 5. “3 less than x” means start at x, take 3 away: x − 3 — watch the order, it is the x that loses 3.

Worked example · Write “7 more than double m” as algebra	
1. Find the pieces	**“double m” and “7 more”**
2. Double m means	**2 × m = 2m**
3. 7 more than that means	**add 7**
4. Write it	**2m + 7**

Build in the order the words act: first the doubling, then the adding. 2m + 7 and 2(m + 7) are different expressions — the words tell you which.

PRACTICE · WRITE AS ALGEBRA · 10 QUESTIONS

1. **5 more than n**	6. **the sum of a and b**
2. **3 less than x**	7. **10 minus y**
3. **double m**	8. **7 more than double n**
4. **half of t**	9. **3 less than 5 times x**
5. **4 times k**	10. **1 more than a quarter of p**

2 Algebra shorthand

Why we never write the × sign

Algebra drops the multiplication sign: 3 × n is written 3n, a × b is written ab, and n × n is written n^2. Division becomes a fraction: k ÷ 4 is written k/4. Numbers always go in front of letters: 3n, never n3.

Worked example · Write 2 × a × b + a × a in shorthand	
1. Drop the × signs in 2 × a × b	**2ab**
2. a × a is a squared	**a^2**
3. Put it together	**$2ab + a^2$**

a × a = a^2, not 2a. Doubling and squaring are different: 2a means two copies added; a^2 means a multiplied by itself.

PRACTICE · WRITE IN SHORTHAND · 10 QUESTIONS

1. **3 × n**	6. **2 × a × b**
2. **a × b**	7. **5 × m × m**
3. **n × n**	8. **(a + b) ÷ 2**
4. **k ÷ 4**	9. **x × y × 3**
5. **1 × x**	10. **n × n × n**

3 Substituting a number

The letter leaves, the number takes its place

To substitute, rewrite the expression with the number in the letter's place — brackets help — then calculate. If n = 4, then 3n + 2 becomes 3 × (4) + 2 = 14. Never glue digits together: 3n with n = 4 is 12, not 34.

Worked example · Evaluate 5n − 3 when n = 6	
1. Write the expression	**5n − 3**
2. Replace n with (6)	**5 × (6) − 3**
3. Multiply first (BIDMAS)	**30 − 3**
4. Finish	**27**

Always write the substitution line “5 × (6) − 3” before calculating. One extra line, and sign and order errors almost disappear.

PRACTICE · EVALUATE · 10 QUESTIONS

1. **3n + 2 when n = 4**	6. $\frac{n}{2}$ **+ 3 when n = 8**
2. **2x + 7 when x = 5**	7. **7a − 10 when a = 2**
3. **5m − 3 when m = 6**	8. **20 − 3y when y = 5**
4. **10 − 2k when k = 3**	9. **6p + p when p = 3**
5. **4t + 1 when t = 0**	10. $\frac{x}{4}$ **+ 9 when x = 12**

4 Two letters at once

Each letter gets its own number

With two letters, substitute both at the same time and keep them apart: if a = 5 and b = 2, then 2a + 3b = 2 × (5) + 3 × (2) = 10 + 6 = 16. Write both substitutions before calculating anything.

Worked example · Evaluate 4x − 2y when x = 3, y = 5	
1. Replace each letter	**4 × (3) − 2 × (5)**
2. Multiply both parts	**12 − 10**
3. Finish	**2**

Two letters, two substitutions, one line each. Do not calculate anything until every letter has been swapped for its number.

PRACTICE · EVALUATE · 10 QUESTIONS

1. **2a + 3b when a = 5, b = 2**	6. **xy when x = 6, y = 7**
2. **4x − 2y when x = 3, y = 5**	7. **2ab when a = 3, b = 4**
3. **a + 2b when a = 7, b = 4**	8. **10 − a − b when a = 3, b = 5**
4. **5m − n when m = 3, n = 8**	9. **ab − c when a = 4, b = 5, c = 9**
5. **3p + 2q when p = 4, q = 5**	10. $\frac{a + b}{2}$ **when a = 9, b = 5**

5 Negative numbers

Brackets are your seatbelt

When the number is negative, ALWAYS put it in brackets before calculating. If $n = -3$, then $n^2 = (-3)^2 = 9$, and $2n = 2 \times (-3) = -6$. Without the brackets, minus signs get lost — with them, the arithmetic runs itself.

Worked example · Evaluate $10 - 2n$ when $n = -4$	
1. Replace n with (-4)	$10 - 2 \times (-4)$
2. Multiply first	$2 \times (-4) = -8$
3. Subtracting -8 means adding 8	$10 - (-8) = 10 + 8$
4. Finish	**18**

Two minus signs met and became a plus. That only stays visible because the brackets kept the -4 in one piece.

PRACTICE · EVALUATE · 10 QUESTIONS

1. **3n when n = -2**	6. **4m − 3 when m = -2**
2. **n + 7 when n = -3**	7. **$n^2 + 1$ when $n = -5$**
3. **n^2 when $n = -3$**	8. **6 − 2k when k = -3**
4. **2x + 1 when x = -5**	9. **ab when a = -4, b = 3**
5. **10 − n when n = -4**	10. **$n^2 - n$ when $n = -2$**

6 BIDMAS inside expressions

$3n^2$ and $(3n)^2$ are different animals

BIDMAS still rules after substitution. In $3n^2$, only n is squared: if $n = 2$, that is $3 \times 4 = 12$. In $(3n)^2$, the whole 3n is squared: $(6)^2 = 36$. Brackets first, then indices, then multiply/divide, then add/subtract.

Worked example · Evaluate 2(n + 3) when n = 5	
1. Replace n with (5)	**2 × (5 + 3)**
2. Brackets first	**5 + 3 = 8**
3. Multiply	**2 × 8 = 16**

The bracket is a package: everything inside gets finished before anything outside touches it.

PRACTICE · EVALUATE · 10 QUESTIONS

1. **2(n + 3) when n = 5**	6. **2m² when m = 3**
2. **3n² when n = 2**	7. **(2m)² when m = 3**
3. **(3n)² when n = 2**	8. **10 − (k + 3) when k = 4**
4. **5(x − 1) when x = 4**	9. **4(t + 2) − 5 when t = 3**
5. **n² + 2n when n = 3**	10. **(n + 1)² when n = 4**

THE SUBSTITUTION ROUTINE

Same four moves, every single time.

1 **Write the expression first**
Copy it down exactly. No shortcuts yet.

2 **Swap every letter for its number — in brackets**
One line: $5n - 3$ becomes $5 \times (6) - 3$. Negative numbers keep their brackets.

3 **Calculate with BIDMAS**
Brackets, indices, multiply/divide, add/subtract.

✓ **Sense-check the answer**
Positive when you expected negative? Huge when inputs were small? Re-check step 2.

THE MISTAKES THAT COST THE MOST MARKS

1. Gluing digits together

WRONG $n = 4$, so $2n = 24$

RIGHT $n = 4$, so $2n = 2 \times 4 = 8$

$2n$ always means $2 \times n$. The invisible multiplication sign is the single most important thing in algebra notation.

2. Confusing $2a$ with a^2

WRONG $a = 5$, so $a^2 = 10$

RIGHT $a = 5$, so $a^2 = 5 \times 5 = 25$

$2a$ is a doubled; a^2 is a times itself. For $a = 5$ they give 10 and 25 — not even close.

3. Losing the minus when squaring

WRONG $n = -3$, so $n^2 = -9$

RIGHT $n = -3$, so $n^2 = (-3)^2 = 9$

Squaring a negative gives a positive: $(-3) \times (-3) = 9$. Brackets around the substituted number make this automatic.

4. Squaring more than you should

WRONG $n = 2$, so $3n^2 = 36$

RIGHT $n = 2$, so $3n^2 = 3 \times 2^2 = 3 \times 4 = 12$

The little ^{2} touches only what it sits on. Only $(3n)^2$ squares the whole thing.

5. Reversing “less than”

WRONG	“3 less than x” → 3 − x
RIGHT	“3 less than x” → x − 3

It is the x that loses 3. Try it with numbers: 3 less than 10 is 7, which is 10 − 3.

TASKS TO SUBMIT

CRITERION B · INVESTIGATING PATTERNS

The Number-Trick Machine

Here is a number trick. Think of a number. Add 4. Double the result. Subtract 8. Halve the result. Try it — you always end up back at your starting number.

1. Run the trick for at least six starting numbers, including 0 and a negative number. Record the results in a table.

2. Describe what you notice.

3. Let the starting number be n. Write what each step does to n, one line per step.

4. Use your algebra to prove the trick always returns n.

5. Now design your own trick that always ends on 5, whatever number the player thinks of. Write it as instructions, test it on three numbers, and prove it with algebra.

Submit: photos or a scan of your working, or a typed document. Marked out of 8 against Criterion B. Testing many numbers is not enough for the top band — the algebra proof is what shows the trick can NEVER fail.

CRITERION D · APPLYING MATHS IN REAL-LIFE CONTEXTS

The Gaborone Taxi

A Gaborone taxi driver charges a P30 pick-up fee plus P7 for every kilometre. The fare in pula for a journey of k kilometres is 30 + 7k.

1. Explain what the 30 and the 7 each mean in real life.

2. Use substitution to find the fare for journeys of 3 km, 8 km and 10 km.

3. A passenger pays P100. Roughly how far did they travel? Show your reasoning.

4. Write the fare expression for a different driver who charges P20 pick-up and P9 per kilometre. Which driver is cheaper for a 4 km trip? For a 12 km trip?

5. Give two real-life situations where the model 30 + 7k would give the wrong fare, and explain why.

Submit: your working plus short written explanations. Marked out of 8 against Criterion D. The top band needs part 5 done properly — a real discussion of where the model breaks, not just correct arithmetic.

ANSWER KEY

Check your answer against the key, and if it differs, find the exact line where your working went a different way.

1. Saying it in symbols

1	$n + 5$	6	$a + b$
2	$x - 3$	7	$10 - y$
3	$2m$	8	$2n + 7$
4	$\frac{t}{2}$	9	$5x - 3$
5	$4k$	10	$\frac{p}{4} + 1$

2. Algebra shorthand

1	$3n$	6	$2ab$
2	ab	7	$5m^2$
3	n^2	8	$\frac{a + b}{2}$
4	$\frac{k}{4}$	9	$3xy$
5	x	10	n^3

3. Substituting a number

1	14	6	7
2	17	7	4
3	27	8	5
4	4	9	21
5	1	10	12

4. Two letters at once

1	16	6	42
2	2	7	24
3	15	8	2
4	7	9	11
5	22	10	7

5. Negative numbers			
1	-6	6	-11
2	4	7	26
3	9	8	12
4	-9	9	-12
5	14	10	6

6. BIDMAS inside expressions			
1	16	6	18
2	12	7	36
3	36	8	3
4	15	9	15
5	15	10	25