ALGEBRA · UNIT 11

The Power Rules

Indices are shorthand for repeated multiplication — and the rules for handling them are not arbitrary. Every single one comes from counting how many copies you have. Count properly and you will never mix them up.

Standard · MYP 4–5 · ~40 minutes · 60 practice questions · Criteria A · B · C · D

BY THE END OF THIS UNIT YOU CAN

- ✓ Say what an index actually means and evaluate simple powers
- ✓ Multiply powers of the same base by adding indices
- ✓ Divide powers of the same base by subtracting indices
- ✓ Raise a power to a power by multiplying indices
- ✓ Explain why $x^0 = 1$ and what a negative index means
- ✓ Handle powers that carry number coefficients

THE ONE IDEA BEHIND ALL OF IT

x^5 means five x's multiplied together. So $x^5 \times x^3$ is five x's next to three x's — eight x's altogether. You are not memorising a rule, you are counting:

$$x^a \times x^b = x^{a+b} \quad x^a \div x^b = x^{a-b} \quad (x^a)^b = x^{ab}$$

Multiply → add the indices. Divide → subtract. Power of a power → multiply.

One warning that saves more marks than any other: these rules only work when the bases are the same. $x^5 \times y^3$ stays x^5y^3 — you cannot combine them, because you are not counting the same thing.

1 What an index means

The small number counts the copies

In 2^5 the 2 is the base and the 5 is the index (or power). It means $2 \times 2 \times 2 \times 2 \times 2 = 32$. The index counts how many copies are multiplied — it does NOT mean multiply by 5.

Worked example · Evaluate 3^4	
1. The base is 3, the index is 4	**so four 3's multiplied**
2. Write it out	**$3 \times 3 \times 3 \times 3$**
3. Work left to right	**$9 \times 3 = 27$, then $27 \times 3 = 81$**
4. Answer	**81**

3^4 is 81, not 12. Multiplying four 3's is very different from 3×4 — and the gap grows fast as the index climbs.

PRACTICE · EVALUATE · 10 QUESTIONS

1. 2^3	6. 4^3
2. 3^2	7. 3^4
3. 5^2	8. 7^2
4. 10^3	9. 2^{10}
5. 2^5	10. 10^6

2 Multiplying powers

Same base → add the indices

$x^5 \times x^3$ means five x's times three x's — eight x's in total, so x^8. Add the indices. This works ONLY when the bases match: $x^5 \times y^3$ cannot be combined at all.

Worked example · Simplify $a^4 \times a^6$	
1. Count the a's	**four a's and six a's**
2. Altogether	**4 + 6 = 10 a's**
3. Write it	**a^{10}**

Add, do not multiply. $a^4 \times a^6$ is a^{10}, not a^{24} — you are joining two piles of a's, not making a pile of piles.

PRACTICE · SIMPLIFY · 10 QUESTIONS

1. $x^5 \times x^3$	6. $p^6 \times p$
2. $a^4 \times a^6$	7. $x^2 \times x^3 \times x^4$
3. $y^2 \times y^7$	8. $k^8 \times k^2$
4. $m^3 \times m^3$	9. $n^5 \times n^5$
5. tt^5	10. $w^3 \times w^4 \times w$

3 Dividing powers

Same base → subtract the indices

$x^7 \div x^3$ means seven x's on top and three underneath. Three cancel from each, leaving four: x^4. Subtract the indices — top minus bottom, in that order.

Worked example · Simplify $a^9 \div a^4$	
1. Nine a's on top, four underneath	**four pairs cancel**
2. What is left on top	**9 − 4 = 5 a's**
3. Write it	a^5

Top minus bottom, always in that order. $a^4 \div a^9$ would give a^{-5} — a perfectly good answer, which Concept 5 explains.

PRACTICE · SIMPLIFY · 10 QUESTIONS

1. $\frac{x^7}{x^3}$	6. $\frac{p^{12}}{p^7}$
2. $\frac{a^9}{a^4}$	7. $\frac{k^5}{k^4}$
3. $\frac{y^6}{y^2}$	8. $\frac{n^{20}}{n^{11}}$
4. $\frac{m^{10}}{m^5}$	9. $\frac{w^6}{w^3}$
5. $\frac{t^8}{t}$	10. $\frac{z^{15}}{z^{10}}$

4 A power of a power

Multiply the indices

$(x^2)^3$ means $x^2 \times x^2 \times x^2$ — three lots of two x's, so six x's: x^6. Multiply the indices. Compare carefully with Concept 2: $x^2 \times x^3$ adds to x^5, but $(x^2)^3$ multiplies to x^6.

Worked example · Simplify $(a^4)^5$	
1. The bracket is repeated 5 times	$a^4 \times a^4 \times a^4 \times a^4 \times a^4$
2. Each bracket brings 4 a's	**5 lots of 4 = 20**
3. Write it	a^{20}

Ask yourself: is this two piles joined (add) or one pile repeated (multiply)? Brackets mean repeated — so multiply.

PRACTICE · SIMPLIFY · 10 QUESTIONS

1. $(x^2)^3$	6. $(p^6)^2$
2. $(a^4)^5$	7. $(k^3)^3$
3. $(y^3)^2$	8. $(n)^8$
4. $(m^5)^3$	9. $(w^{10})^2$
5. $(t^2)^7$	10. $(z^4)^4$

5 Zero and negative indices

Where the pattern leads

Follow the pattern down: $2^3 = 8$, $2^2 = 4$, $2^1 = 2$ — each step halves. One more step gives $2^0 = 1$, and the next gives $2^{-1} = ½$. So a negative index means one over: $x^{-2} = 1/x^2$.

Worked example · Why does $x^0 = 1$?	
1. Take any power divided by itself	$x^5 \div x^5$
2. Anything divided by itself is	1
3. But the index rule says subtract	$x^{5-5} = x^0$
4. Both must be true, so	$x^0 = 1$

$x^0 = 1$ is not a strange exception someone invented. It is forced — any other value would break the division rule.

PRACTICE · SIMPLIFY · 10 QUESTIONS

1. 5^0	6. 10^{-2}
2. 12^0	7. x^{-2}
3. x^0	8. a^{-1}
4. 2^{-1}	9. 3^{-2}
5. 2^{-3}	10. $\frac{x^3}{x^7}$

6 Powers with coefficients

Numbers together, letters together

In $5x^3 \times 3x^4$, deal with the two jobs separately: multiply the numbers ($5 \times 3 = 15$) and then the letters ($x^3 \times x^4 = x^7$). Answer $15x^7$. Same split for dividing.

Worked example · Simplify $12a^7 \div 4a^3$	
1. Numbers first	$12 \div 4 = 3$
2. Then letters	$a^7 \div a^3 = a^4$
3. Put them together	$3a^4$

Two separate jobs, done in order. Trying to handle the numbers and the indices in one move is where the errors live.

PRACTICE · SIMPLIFY · 10 QUESTIONS

1. $5x^3 \times 3x^4$	6. $(2x^3)^2$
2. $2a^2 \times 6a^5$	7. $(3a^2)^2$
3. $4y \times 5y^3$	8. $7m^4 \times 2m$
4. $\frac{12a^7}{4a^3}$	9. $\frac{18p^8}{6p^5}$
5. $\frac{20x^6}{5x^2}$	10. $(5t^2)^2$

WHICH POWER RULE DO I USE?

Look at what is happening BETWEEN the powers.

× **Multiplying powers of the same base?**
ADD the indices. $x^5 \times x^3 = x^8$.

÷ **Dividing powers of the same base?**
SUBTRACT, top minus bottom. $x^7 \div x^3 = x^4$.

( ) **A power raised to another power?**
MULTIPLY the indices. $(x^2)^3 = x^6$.

✓ **Different bases, or a coefficient in front?**
Different bases never combine. Coefficients are a separate job — numbers with numbers, letters with letters.

THE MISTAKES THAT COST THE MOST MARKS

1. Multiplying the index instead of counting copies

WRONG $3^4 = 12$

RIGHT $3^4 = 3 \times 3 \times 3 \times 3 = 81$

The index counts copies; it is not a multiplier. The two answers differ by a factor of nearly seven here, and far more for bigger indices.

2. Multiplying indices when you should add

WRONG $x^5 \times x^3 = x^{15}$

RIGHT $x^5 \times x^3 = x^8$

Joining two piles of x's ADDS them. Multiplying indices is for $(x^5)^3$, which is a pile repeated.

3. Combining different bases

WRONG $x^5 \times y^3 = xy^8$

RIGHT $x^5 \times y^3 = x^5y^3$ — it cannot be simplified

You can only count things that are the same. Five x's and three y's are different objects.

4. Thinking $x^0 = 0$

WRONG $x^0 = 0$

RIGHT $x^0 = 1$

Follow the halving pattern: $2^3 = 8$, $2^2 = 4$, $2^1 = 2$, $2^0 = 1$. Or use $x^5 \div x^5 = 1$ and the subtraction rule. Both force the answer to be 1.

5. Making a negative index give a negative answer

WRONG $2^{-3} = -8$

RIGHT $2^{-3} = 1/8$

A negative index means RECIPROCAL, not negative. The answer of a negative power of a positive number is always positive.

TASKS TO SUBMIT

CRITERION B · INVESTIGATING PATTERNS

The Zero-Power Mystery

Most students are simply told that $x^0 = 1$ and asked to accept it. In this investigation you are going to discover why it has to be true — and prove that no other answer is possible.

1. Write out the values of 2^5, 2^4, 2^3, 2^2, 2^1 in a table. Describe exactly what happens to the value each time the index drops by 1.

2. Continue your pattern for two more steps to predict 2^0 and 2^{-1}. State what the pattern forces those values to be.

3. Repeat the whole investigation with base 3 and base 10. Does the same thing happen? What is the general result?

4. Now prove it a second, independent way: use $x^a \div x^a$. Write down what the answer must be by common sense, then what the subtraction rule gives, and explain why the two together force $x^0 = 1$.

5. Finally, classify this claim as always / sometimes / never true: "$x^0 = 1$". Investigate the single value of x that causes trouble and explain what goes wrong there.

Submit: your tables, both proofs and your written reasoning. Marked out of 8 against Criterion B. Spotting the pattern reaches the middle bands; the top band needs BOTH justifications in part 4 and a correct treatment of the awkward case in part 5.

CRITERION D · APPLYING MATHS IN REAL-LIFE CONTEXTS

The Chain Message

A WhatsApp message asks each person to forward it to 3 new people. One person starts it. In round 1, 3 people receive it; in round 2, each of those forwards to 3 more, and so on. Botswana's population is about 2.4 million.

1. Write an expression for the number of people receiving the message in round n, and use it to complete a table for rounds 1 to 10.

2. Describe how fast the numbers grow. Compare round 10 with round 5 — how many times bigger is it, and why is that number itself a power of 3?

3. Find the first round in which the number of new recipients would exceed Botswana's population. Show your working.

4. Your model says the whole country is reached in a fortnight of forwarding. In reality chain messages never do this. Give three specific reasons the model overstates the spread.

5. Suggest one change to the model that would make it more realistic, and explain what mathematical effect your change has on the growth.

Submit: your table, working and written explanations. Marked out of 8 against Criterion D. Parts 1–3 are the arithmetic; parts 4–5 are where the top band is decided — a model this wrong about reality is only useful if you can say WHY.

ANSWER KEY

Check your answer against the key, and if it differs, find the exact line where your working went a different way.

1. What an index means

1	**8**	6	**64**
2	**9**	7	**81**
3	**25**	8	**49**
4	**1000**	9	**1024**
5	**32**	10	**1000000**

2. Multiplying powers

1	**x^{8}**	6	**p^{7}**
2	**a^{10}**	7	**x^{9}**
3	**y^{9}**	8	**k^{10}**
4	**m^{6}**	9	**n^{10}**
5	**t^{6}**	10	**w^{8}**

3. Dividing powers

1	**x^{4}**	6	**p^{5}**
2	**a^{5}**	7	**k**
3	**y^{4}**	8	**n^{9}**
4	**m^{5}**	9	**w^{3}**
5	**t^{7}**	10	**z^{5}**

4. A power of a power

1	**x^{6}**	6	**p^{12}**
2	**a^{20}**	7	**k^{9}**
3	**y^{6}**	8	**n^{8}**
4	**m^{15}**	9	**w^{20}**
5	**t^{14}**	10	**z^{16}**

5. Zero and negative indices

1	1	6	$\frac{1}{100}$
2	1	7	$\frac{1}{x^2}$
3	1	8	$\frac{1}{a}$
4	$\frac{1}{2}$	9	$\frac{1}{9}$
5	$\frac{1}{8}$	10	$\frac{1}{x^4}$

6. Powers with coefficients

1	$15x^7$	6	$4x^6$
2	$12a^7$	7	$9a^4$
3	$20y^4$	8	$14m^5$
4	$3a^4$	9	$3p^3$
5	$4x^4$	10	$25t^4$