ALGEBRA · UNIT 19

Root and Branch: Surds

√2 is not 1.41. It is not 1.414213 either. Every decimal you write down is slightly wrong — so mathematicians keep the root symbol and stay exactly right. Working with surds is working with perfect precision.

Extended · MYP 4–5 · ~40 minutes · 60 practice questions · Criteria A · B · C · D

BY THE END OF THIS UNIT YOU CAN

- ✓ Say what a surd is and why we keep it exact
- ✓ Simplify √n by hunting the square factor
- ✓ Multiply and divide surds with the √(ab) law
- ✓ Add and subtract like surds
- ✓ Expand brackets containing surds
- ✓ Rationalise a simple denominator

THE ONE IDEA BEHIND ALL OF IT

One law runs this whole unit — the root of a product is the product of the roots:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b} \text{ so } \sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$$

Split out the biggest square factor; its root walks out of the surd.

One warning before you start: this law works for MULTIPLYING only. √9 + √16 = 3 + 4 = 7, but √25 = 5. There is no addition law — and that single counterexample proves it.

1 What a surd is

Roots that refuse to be whole

√25 = 5 exactly — not a surd. √24 has no exact decimal: it is 4.89897…, going on forever without repeating. A surd is a root left in exact form because any decimal would be an approximation. Know your perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.

Worked example · Which of √36, √40 and √49 are surds?	
1. √36	**= 6 exactly — not a surd**
2. √49	**= 7 exactly — not a surd**
3. √40	**= 6.324… never ends — a surd**
4. Conclusion	**only √40 is a surd**

Between 36 and 49 every root is a surd. Perfect squares are rare — most roots are surds, which is why this unit exists.

PRACTICE · EVALUATE OR KEEP EXACT · 10 QUESTIONS

1. **√25**	6. **√4 × √9**
2. **√64**	7. **Is √30 a surd?**
3. **√121**	8. **Is √81 a surd?**
4. **√144**	9. **Is √50 a surd?**
5. **√400**	10. **Is √169 a surd?**

2 Simplifying √n

Hunt the square factor

Split n into (biggest square factor) × (what is left). The square's root walks outside; the rest stays under the root. √50 = √25 × √2 = 5√2. Always take the BIGGEST square factor — √50 = √4 × √12.5 is true but useless.

Worked example · Simplify √72	
1. Hunt the biggest square factor of 72	**36 × 2 = 72, and 36 = 6²**
2. Split the root	**√72 = √36 × √2**
3. Take the root of the square	**6√2**
4. Check on a calculator	**√72 = 8.485… and 6√2 = 8.485… ✓**

If you had picked 9 × 8 you would get 3√8 — not finished, because 8 still hides a square. Biggest square first, always.

PRACTICE · SIMPLIFY · 10 QUESTIONS

1. **√8**	6. **√32**
2. **√12**	7. **√50**
3. **√18**	8. **√72**
4. **√20**	9. **√75**
5. **√27**	10. **√200**

3 Multiplying and dividing surds

The law works both ways

√a × √b = √(ab): multiply under one root, then simplify. √2 × √8 = √16 = 4 — two surds multiplying to a whole number. Division matches: √a ÷ √b = √(a/b), so √48 ÷ √3 = √16 = 4.

Worked example · Simplify √6 × √8	
1. Multiply under one root	**√6 × √8 = √48**
2. Hunt the biggest square factor	**48 = 16 × 3**
3. Split and simplify	**√16 × √3 = 4√3**

Multiply first, simplify second. Trying to simplify √6 and √8 separately first also works (√8 = 2√2, then √6 × 2√2 = 2√12 = 4√3) but takes twice the steps.

PRACTICE · SIMPLIFY · 10 QUESTIONS

1. **√2 × √8**	6. **√3 × √6**
2. **√3 × √12**	7. **√10 × √5**
3. **√2 × √18**	8. **√48 ÷ √3**
4. **√5 × √20**	9. **√50 ÷ √2**
5. **√6 × √8**	10. **√72 ÷ √8**

4 Adding and subtracting surds

Like surds only — Unit 2's rule returns

3√5 + 2√5 = 5√5, exactly like 3a + 2a = 5a. But 3√5 + 2√3 stays as it is — different surds never combine, just like different letters. Sometimes simplifying FIRST reveals hidden like surds: √8 + √2 = 2√2 + √2 = 3√2.

Worked example · Simplify √50 + √18	
1. They look different — simplify each first	**√50 = 5√2 and √18 = 3√2**
2. Now they are LIKE surds	**5√2 + 3√2**
3. Count them, Unit 2 style	**8√2**

√50 + √18 ≠ √68. The addition trap again — always simplify each surd first and see whether matching surds appear.

PRACTICE · SIMPLIFY · 10 QUESTIONS

1. **3√5 + 2√5**	6. **√50 + √18**
2. **7√2 − 3√2**	7. **√27 + √12**
3. **4√3 + √3**	8. **√75 − √27**
4. **6√7 − 6√7**	9. **√72 − √32**
5. **√8 + √2**	10. **2√20 + √45**

5 Brackets with surds

Unit 3's arrows, exact numbers

Expand exactly as in Unit 3 — every term inside gets multiplied — remembering √2 × √2 = 2. So √2(√2 + 3) = 2 + 3√2. Double brackets use the same four arrows as Unit 13.

Worked example · Expand (1 + √2)(3 − √2)	
1. First × first	**1 × 3 = 3**
2. Outer	**1 × (−√2) = −√2**
3. Inner	**√2 × 3 = 3√2**
4. Last	**√2 × (−√2) = −2**
5. Collect	**3 − 2 − √2 + 3√2 = 1 + 2√2**

√2 × √2 = 2, not 4 and not √4-left-alone. The root and the square cancel — that is what a square root IS.

PRACTICE · EXPAND AND SIMPLIFY · 10 QUESTIONS

1. **√2(√2 + 3)**	6. **(2 + √3)(2 − √3)**
2. **√3(√3 − 2)**	7. **(1 + √5)²**
3. **√5(2√5 + 1)**	8. **(3 + √2)(3 − √2)**
4. **2√2(3√2 − 1)**	9. **(√7 + 1)(√7 − 1)**
5. **(1 + √2)(3 − √2)**	10. **(4 − √3)(4 + √3)**

6 Clearing the root downstairs

Multiply by a clever form of 1

Convention says a surd may not live in the denominator. Fix it by multiplying top AND bottom by that surd — which is multiplying by 1, so the VALUE never changes, only the look: 1/√2 = √2/2.

Worked example · Rationalise $\frac{6}{\sqrt{3}}$	
1. Multiply top and bottom by $\sqrt{3}$	$\frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$
2. The bottom becomes whole	$\sqrt{3} \times \sqrt{3} = 3$
3.	$\frac{6\sqrt{3}}{3}$
4. Simplify the fraction	$2\sqrt{3}$

You multiplied by $\frac{\sqrt{3}}{\sqrt{3}} = 1$, so nothing changed except the form. Check: $6 \div 1.732 \approx 3.46$ and $2 \times 1.732 \approx 3.46$ ✓.

PRACTICE · RATIONALISE THE DENOMINATOR · 10 QUESTIONS

1. $\frac{1}{\sqrt{2}}$	6. $\frac{4}{\sqrt{2}}$
2. $\frac{1}{\sqrt{3}}$	7. $\frac{10}{\sqrt{5}}$
3. $\frac{2}{\sqrt{2}}$	8. $\frac{3}{\sqrt{6}}$
4. $\frac{6}{\sqrt{3}}$	9. $\frac{12}{\sqrt{3}}$
5. $\frac{5}{\sqrt{5}}$	10. $\frac{\sqrt{2}}{\sqrt{8}}$

THE SURD ROUTINE

One law, four habits.

1 **Hunt the biggest square factor**
√n simplifies exactly when n hides a square: 4, 9, 16, 25, 36, 49, 64, 81, 100, 144.

2 **Multiplying? Merge under one root first**
√a × √b = √(ab), then simplify the result.

3 **Adding? Only LIKE surds combine**
Simplify each surd first — hidden matches often appear. Never add under the root.

✓ **Decimal-check on a calculator**
√72 and 6√2 must show the same decimal. Ten seconds, and it catches every wrong square factor.

THE MISTAKES THAT COST THE MOST MARKS

1. Adding under the root

WRONG $\sqrt{9} + \sqrt{16} = \sqrt{25} = 5$

RIGHT $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$

The √(ab) law is for MULTIPLYING only. This one counterexample is worth memorising whole — it kills the error forever.

2. Not taking the biggest square factor

WRONG $\sqrt{72} = \sqrt{9} \times \sqrt{8} = 3\sqrt{8}$ — and stopping

RIGHT $\sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

3√8 is not finished: 8 still hides a 4. Fully simplified means nothing square left under the root — the Unit 4 rule in new clothes.

3. √2 × √2 = 4

WRONG $\sqrt{2} \times \sqrt{2} = 4$

RIGHT $\sqrt{2} \times \sqrt{2} = 2$

The root of 2, squared, gives back 2 — rooting and squaring undo each other. Getting 4 means you squared the 2 as well.

4. Combining unlike surds

WRONG $3\sqrt{2} + 2\sqrt{3} = 5\sqrt{5}$

RIGHT $3\sqrt{2} + 2\sqrt{3}$ cannot be simplified

√2 and √3 are different objects, like a and b in Unit 2. Neither the counts nor the radicands add.

5. Rationalising only the bottom

WRONG $1/\sqrt{2} = 1/2$

RIGHT $1/\sqrt{2} = \sqrt{2}/2$

Multiply top AND bottom by √2. Multiplying only the bottom changes the value — check: $1/\sqrt{2} \approx 0.707$ but $1/2 = 0.5$.

TASKS TO SUBMIT

CRITERION B · INVESTIGATING PATTERNS

The Surd Multiplication Law

You have used $\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$ all unit. But is it actually TRUE — and how would you know? Testing examples builds confidence; only a proof settles it. This investigation does both, then finds the law that ISN'T true.

1. Test $\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$ on five different pairs (a, b), using a calculator to compare decimals. Include one pair where both roots are surds and one where the product comes out whole.

2. Now prove it: let $x = \sqrt{a} \times \sqrt{b}$. Square x, using $(pq)^2 = p^2q^2$. What does x^2 equal, and why does that force $x = \sqrt{(ab)}$?

3. Use the law to explain WHY the simplification method works: show the chain $\sqrt{72} \to \sqrt{(36 \times 2)} \to \sqrt{36} \times \sqrt{2} \to 6\sqrt{2}$, naming the law at the step it is used.

4. Now investigate the imposter: is $\sqrt{(a + b)} = \sqrt{a} + \sqrt{b}$ ever true? Test five pairs. Find ONE counterexample and explain why one is enough to kill the general claim.

5. Harder: find ALL the pairs for which $\sqrt{(a + b)} = \sqrt{a} + \sqrt{b}$ IS true. (Square both sides and see what the algebra forces.)

Submit: your tests, proof and reasoning. Marked out of 8 against Criterion B. The squaring proof in part 2 reaches the upper band; part 5's complete answer — the equation forces $2\sqrt{(ab)} = 0$, so $a = 0$ or $b = 0$ — is the top.

CRITERION D · APPLYING MATHS IN REAL-LIFE CONTEXTS

The TV in the Shop Window

A Gaborone electronics shop advertises a “50-inch” TV. That number is the DIAGONAL, corner to corner. Modern screens have sides in the ratio 16 : 9 — so the width is 16k and the height 9k for some scale factor k.

1. Use Pythagoras on the 16k by 9k screen to show that the diagonal is $k\sqrt{337}$, keeping your working exact (no decimals yet).

2. For the 50-inch TV, find k exactly, then the width and height exactly — each as a multiple of $1/\sqrt{337}$ — and only THEN as decimals to 1 d.p.

3. Verify your answer the surd way: show that $\text{width}^2 + \text{height}^2$ gives exactly 2500, using fractions — no rounding anywhere.

4. A rival shop offers a 40-inch TV. Without repeating Pythagoras, use proportion (Unit 17!) to write down its width and height, and find the ratio of the two screen AREAS as an exact fraction.

5. The salesman says “50 inches is 25% more screen than 40 inches.” Use your area ratio to decide whether that claim understates or overstates the difference, and explain the general rule connecting diagonal ratio to area ratio.

Submit: your exact working, decimal conversions and written explanations. Marked out of 8 against Criterion D. The point of the task is that EXACT surd arithmetic proves things decimals cannot — part 3 works to the last digit only because nothing was rounded.

ANSWER KEY

Check your answer against the key, and if it differs, find the exact line where your working went a different way.

1. What a surd is

1	5	6	6
2	8	7	yes — 30 is not a perfect square
3	11	8	no — √81 = 9 exactly
4	12	9	yes — but it simplifies to 5√2
5	20	10	no — √169 = 13 exactly

2. Simplifying √n

1	2√2	6	4√2
2	2√3	7	5√2
3	3√2	8	6√2
4	2√5	9	5√3
5	3√3	10	10√2

3. Multiplying and dividing surds

1	4	6	3√2
2	6	7	5√2
3	6	8	4
4	10	9	5
5	4√3	10	3

4. Adding and subtracting surds

1	5√5	6	8√2
2	4√2	7	5√3
3	5√3	8	2√3
4	0	9	2√2
5	3√2	10	7√5

5. Brackets with surds			
1	$2 + 3\sqrt{2}$	6	1
2	$3 - 2\sqrt{3}$	7	$6 + 2\sqrt{5}$
3	$10 + \sqrt{5}$	8	7
4	$12 - 2\sqrt{2}$	9	6
5	$1 + 2\sqrt{2}$	10	13

6. Clearing the root downstairs			
1	$\frac{\sqrt{2}}{2}$	6	$2\sqrt{2}$
2	$\frac{\sqrt{3}}{3}$	7	$2\sqrt{5}$
3	$\sqrt{2}$	8	$\frac{\sqrt{6}}{2}$
4	$2\sqrt{3}$	9	$4\sqrt{3}$
5	$\sqrt{5}$	10	$\frac{1}{2}$